Measuring Productivity Changes of Hospitals in Tehran: The Fuzzy Malmquist Productivity Index

Pejman Peykani[1], Fatemeh Sadat Seyed Esmaeili[2],

Mohsen Rostamy-Malkhalifeh[2,*], Farhad Hosseinzadeh Lotfi[2]

[1] School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

[2] Faculty of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

Abstract

Background and Objectives: The purpose of this paper is to suggest a novel method to measure the productivity changes of hospitals over time in the presence of linguistic variables along with fuzzy data.

Methods: Applying the popular and applicable approaches including data envelopment analysis (DEA), Malmquist productivity index (MPI) and possibilistic programming, the fuzzy Malmquist productivity index (FMPI) is proposed.

Results: In this study, the proposed fuzzy MPI is implemented for measuring productivity changes of 10 hospitals in Tehran. Notably, the input variables include the number of beds, the number of doctors, equipment & infrastructures and hospital location. Also, the output variables include the number of inpatient days, the number of outpatient, and overall patient satisfaction. According to the obtained results, the productivity of 5 hospitals has increased in 2014 in comparison to 2013.

Conclusions: The obtained results have shown the capability of the proposed index to calculate the changes in productivity of hospitals in the presence of ambiguity in data.

Keywords: Hospital Productivity, Malmquist Productivity Index, Data Envelopment Analysis, Fuzzy Mathematical Programming, Fuzzy Data.

1. Introduction

Assessing the performance and productivity of health care systems and treatment centers such as hospitals is of great important and almost mandatory. Data envelopment analysis (DEA) is one of the effective and popular methods that widely used in health care.[1-3] DEA is a powerful mathematical programming approach that can be applied for performance measurement, and ranking of homogenous decision making units (DMUs).[4-8] Moreover, DEA approach, unlike other multiple criteria decision making (MCDM) techniques, also has the capability of benchmarking for inefficient DMUs.

One of the important issues in the performance evaluating of hospitals in real-world cases and applications is to identify the advance as well as regress of each hospital over time periods. In this regard, it is essential to know whether the hospital has a degree or type of functional change, including progression, regression, or stagnation over its previous period compared to other hospitals. Using DEA and Malmquist productivity index (MPI) enables us to identify, calculate, and evaluate the trends and types of hospital productivity changes.

It should be noted that some of the variables, such as overall patient satisfaction level, which should be considered in performance evaluating of hospitals, are linguistic variables. These linguistic variables can be converted to fuzzy variables. In this respect, it is imperative to say that the very important point that should be taken into account when calculating MPI is to consider the uncertainty and ambiguity in the data.

Classic DEA models cannot encompass the uncertainty of data. As a result, ignoring this point may mislead ones over the identification and classification of DMUs in terms of the process of productivity changes. In this study, the development of novel Malmquist productivity index with the ability to be implemented in the presence of fuzzy data is discussed. For this purpose, the possibilistic programming approach, which is one of the most useful and effective approaches in fuzzy mathematical programming, has been employed. It should be noted that the main research questions of the current study can be mentioned as follows: What is the trend of hospital productivity changes over time periods? How to convert linguistic variables into quantitative concepts in order to measure hospital performance?

The rest of this paper is organized as follows. The literature review, as well as literature gaps, will be introduced in section 2. Section 3 provides the background involving the MPI and fuzzy event via necessity measure. The novel fuzzy MPI approach based on possibilistic programming will be proposed in section 4. Then, the proposed PMPI will be applied for measuring productivity changes of 10 hospitals in a real-life case study of hospitals in Tehran in section 5. Finally, the conclusions and some directions for future research will be given in section 6.

2. Literature Review

In this section, the literature of previous research from two viewpoints including the application of MPI in Iranian hospitals, and the application of uncertain DEA in health care will be reviewed. Then, the literature gaps, which this study aims to fill, are presented.

2.1. The Application of MPI in Iranian Hospitals

In the first viewpoint, all studies that applied MPI for measuring productivity changes of Iranian hospitals have been collected. Table (1) shows the characteristics of these studies as well as current research:

Table (1): The Application of MPI in Iranian Hospitals: A Review

Year	Research	Inputs	Outputs	Uncertainty
2010	Hatam et al.[9]	1. Number of Fixed Beds 2. Number of Nurses 3. Number of Physicians 4. Number of Other Personnels 5. Total Expenses	1. Bed Occupancy Rate 2. Patient–Day Admissions 3. Occupied Bed-Days 4. Average Length of Stay 5. Rate of Bed Turn-Over	-
2014	Lotfi et al.[10]	1. Number of Active Beds 2. Number of Nurses 3. Number of Physicians 4. Number of Other Personnels	1. Bed Occupancy Rate 2. Number of Patients 3. Number of Operations	-
2014	Torabipour et al.[11]	1. Number of Occupied Beds 2. Number of Nurses 3. Number of Physicians	1. Number of Outpatients and Inpatients 2. Average of Hospital Stay 3. Number of Major Operations	-
2017	Raei et al.[12]	1. Number of Beds 2. Number of Physicians 3. Number of Non-Physician Staff	1. Number of Admissions 2. Number of Mortality in Patients	-
2018	Alinezhad & Mirmozaffari[13]	1. Number of Beds 2. Numbers of Nurses and Secretaries 3. Number of Doctors	1. Outpatient Treated 2. Inpatient Treated	-
2019	Our Work	1. Number of Beds 2. Number of Doctors 3. Equipment & Infrastructures 4. Hospital Location	1. Number of Inpatient Days 2. Number of Outpatient 3. Overall Patient Satisfaction	✓

As can be seen in Table (1), all of the existing studies in the literature are neglected the uncertainty of data for measuring productivity changes of Iranian hospitals. However, in the current paper, the productivity changes of hospitals over time can be measured using linguistic variables and fuzzy data.

2.2. The Application of Uncertain DEA in Health Care

In the second viewpoint, all research that considered the uncertainty of data in performance measurement of hospitals have been gathered. Table (2) summarizes the main characteristics of the previous studies and compares them with the FMPI approach that proposed in this paper:

Table (2): The Application of Uncertain DEA in Health Care: A Review

Year	Research	DEA Approach							Form		Uncertainty Method				
		CCR	BCC	SBM	MPI	Congestion	Cost Efficiency	Cross Efficiency	Multiplier	Envelopment	SDEA	FDEA	RDEA	BDEA	IDEA
2012	Ebrahimnejad[14]						✓			✓		✓			
2012	Hatami-Marbini et al.[15]				✓				✓			✓			
2012	Khaki et al.[16]	✓							✓				✓		
2013	Costantino et al.[17]							✓	✓			✓			
2013	De Nicola et al.[18]		✓							✓				✓	
2014	Kalantary & Azar[19]	✓							✓				✓		
2014	Karadayi & Karsak[20]	✓							✓			✓			
2015	Haji Sami et al.[21]	✓							✓				✓		
2015	Kheirollahi et al.[22]					✓				✓	✓				
2015	Mitropoulos et al.[23]	✓								✓	✓				
2016	Rabbani et al.[24]	✓							✓				✓	✓	✓
2017	Arya & Yadav[25]	✓							✓			✓			
2017	Karsak & Karadayi[26]	✓							✓			✓			
2017	Kheirollahi et al.[27]					✓				✓		✓			
2018	Wu & Wu[28]	✓							✓				✓		
2019	Hatefi & Haeri[29]	✓							✓	✓		✓			
2019	Peykani et al.[30]	✓							✓			✓			
2019	Our Work				✓					✓		✓			

It should be noted that in the research conducted by Hatami-Marbini et al.[15], the α-level based approach is applied for dealing with linguistic variables and fuzzy data, while in the current research, the possibilistic programming approach will be applied for proposing FMPI.

3. Background

In this section, the modeling and formulations of Malmquist productivity index based on DEA approach as well as the mathematical formulation of necessity measure to calculate the chances of occurrence of fuzzy events will be discussed.

3.1. Malmquist Productivity Index

Färe & Grosskopf[31] were the pioneer researchers that combined MPI and DEA method to calculate the productivity changes. They have proposed this indicator by taking into account two periods of time and calculating technological changes and efficiency changes over these two periods.

Suppose that there are n homogenous decision making units $DMU_j \ (j=1,\dots,n)$ that convert m input $x_{ij} \ (i=1,\dots,m)$ into s outputs $y_{rj} \ (r=1,\dots,s)$ and DMU_0 is an under evaluation DMU. By applying the envelopment form of input-oriented CCR[5] model, $\Omega_0^t\left(x_0^t, y_0^t\right)$, $\Omega_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right)$, $\Omega_0^t\left(x_0^{t+1}, y_0^{t+1}\right)$, and $\Omega_0^{t+1}\left(x_0^t, y_0^t\right)$ are estimated from Models (1), (2), (3), and (4), respectively.

$$\Omega_0^t\left(x_0^t, y_0^t\right) = \text{Min} \ \ \theta \tag{1-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{i0}^t, \quad \forall i \tag{1-2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{r0}^t, \quad \forall r \tag{1-3}$$

$$\lambda_j \geq 0, \quad \forall j \tag{1-4}$$

$$\Omega_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \ \ \theta \tag{2-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{i0}^{t+1}, \quad \forall i \tag{2-2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{r0}^{t+1}, \quad \forall r \tag{2-3}$$

$$\lambda_j \geq 0, \quad \forall j \tag{2-4}$$

$$\Omega_0^t\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \ \ \theta \tag{3-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t} \leq \theta x_{i0}^{t+1}, \ \ \forall i \tag{3-2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t} \geq y_{r0}^{t+1}, \ \ \forall r \tag{3-3}$$

$$\lambda_j \geq 0, \ \ \forall j \tag{3-4}$$

$$\Omega_0^{t+1}\left(x_0^{t}, y_0^{t}\right) = \text{Min} \ \ \theta \tag{4-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{i0}^{t}, \ \ \forall i \tag{4-2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{r0}^{t}, \ \ \forall r \tag{4-3}$$

$$\lambda_j \geq 0, \ \ \forall j \tag{4-4}$$

Finally, the Malmquist productivity index is calculated using Equation (5):

$$\text{MPI}_0 = \sqrt{\frac{\Omega_0^t\left(x_0^{t+1}, y_0^{t+1}\right) * \Omega_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right)}{\Omega_0^t\left(x_0^{t}, y_0^{t}\right) * \Omega_0^{t+1}\left(x_0^{t}, y_0^{t}\right)}} \tag{5}$$

It needs to be explained that based on the value of the MPI, which can be more or equal to or less than one, the productivity change of the DMU under consideration is interpreted as follows:

- ❒ $\text{MPI}_0 > 1$, the productivity increases and the progress is observed
- ❒ $\text{MPI}_0 < 1$, the productivity decreases and the regress is observed
- ❒ $\text{MPI}_0 = 1$, there is no change in productivity at time $t+1$ in comparison to t.

3.2. Necessity Measure

Suppose that the triple $(\dagger, P(\dagger), Pos)$ be a possibility space where a universe set $\dagger$ is a non-empty set, containing all possible events and $P(\dagger)$ is the power set of $\dagger$. Let $\tilde{\alpha}$ be a triangular fuzzy variable on the mentioned possibility space. The necessity of fuzzy events $\{\tilde{\alpha} \leq \beta\}$ and $\{\tilde{\alpha} \geq \beta\}$ are defined as Equations (6-1) and (6-2):

$$Nec\{\tilde{\alpha} \le \beta\} = \begin{cases} 1, & if \ \alpha^{(3)} \le \beta \, ; \\ \dfrac{\beta - \alpha^{(2)}}{\alpha^{(3)} - \alpha^{(2)}}, & if \ \alpha^{(2)} \le \beta \le \alpha^{(3)} \, ; \\ 0, & if \ \alpha^{(2)} \ge \beta \, . \end{cases} \quad (6\text{-}1)$$

$$Nec\{\tilde{\alpha} \ge \beta\} = \begin{cases} 1, & if \ \alpha^{(1)} \ge \beta \, ; \\ \dfrac{\alpha^{(2)} - \beta}{\alpha^{(2)} - \alpha^{(1)}}, & if \ \alpha^{(1)} \le \beta \le \alpha^{(2)} \, ; \\ 0, & if \ \alpha^{(2)} \le \beta \, . \end{cases} \quad (6\text{-}2)$$

According to necessity measure, converting of fuzzy chance constraints into their equivalent crisp ones in a special confidence level (π) is conducted by Equations (7-1) and (7-2):

$$Nec\{\tilde{\alpha} \le \beta\} \ge \pi \Leftrightarrow (1-\pi)\,\alpha^{(2)} + \pi\,\alpha^{(3)} \le \beta \quad (7\text{-}1)$$

$$Nec\{\tilde{\alpha} \ge \beta\} \ge \pi \Leftrightarrow \pi\,\alpha^{(1)} + (1-\pi)\,\alpha^{(2)} \ge \beta \quad (7\text{-}2)$$

It should be noted that the possibilistic programming approach is an applicable method in fuzzy DEA for dealing with the uncertainty ensue from the absence or lack of knowledge about the exact value of model parameters in fuzzy mathematical programming.[32-37]

4. Fuzzy Malmquist Productivity Index

In this section, the fuzzy Malmquist productivity index (FMPI) is proposed by applying the necessity measure. Note that the inputs and outputs follow a triangular distribution $\tilde{x}(x^{(1)}, x^{(2)}, x^{(3)})$ and $\tilde{y}(y^{(1)}, y^{(2)}, y^{(3)})$ given that $x^{(1)} < x^{(2)} < x^{(3)}$ and $y^{(1)} < y^{(2)} < y^{(3)}$. Now by applying the Equations (7-1) and (7-2), Models (1) to (4) are rewritten to Models (8) to (11), respectively. Note that π is the confidence level for satisfying the fuzzy chance constraints.

$$\Psi_0^t\left(x_0^t, y_0^t, \pi\right) = \text{Min} \ \ \theta \quad (8\text{-}1)$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\pi\right) x_{ij}^{t\,(2)} + \left(\pi\right) x_{ij}^{t\,(3)}\right) \le \theta\left(\left(\pi\right) x_{i0}^{t\,(1)} + \left(1-\pi\right) x_{i0}^{t\,(2)}\right), \ \ \forall i \quad (8\text{-}2)$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\pi\right) y_{rj}^{t\,(1)} + \left(1-\pi\right) y_{rj}^{t\,(2)}\right) \ge \left(\left(1-\pi\right) y_{r0}^{t\,(2)} + \left(\pi\right) y_{r0}^{t\,(3)}\right), \ \ \forall r \quad (8\text{-}3)$$

$$\lambda_j \ge 0, \ \ \forall j \quad (8\text{-}4)$$

$$\Psi_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \;\; \theta \tag{9-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\pi\right) x_{ij}^{t+1(2)} + \left(\pi\right) x_{ij}^{t+1(3)}\right) \le \theta\left(\left(\pi\right) x_{i0}^{t+1(1)} + \left(1-\pi\right) x_{i0}^{t+1(2)}\right), \;\; \forall i \tag{9-2}$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\pi\right) y_{rj}^{t+1(1)} + \left(1-\pi\right) y_{rj}^{t+1(2)}\right) \ge \left(\left(1-\pi\right) y_{r0}^{t+1(2)} + \left(\pi\right) y_{r0}^{t+1(3)}\right), \;\; \forall r \tag{9-3}$$

$$\lambda_j \ge 0, \;\; \forall j \tag{9-4}$$

$$\Psi_0^{t}\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \;\; \theta \tag{10-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\pi\right) x_{ij}^{t(2)} + \left(\pi\right) x_{ij}^{t(3)}\right) \le \theta\left(\left(\pi\right) x_{i0}^{t+1(1)} + \left(1-\pi\right) x_{i0}^{t+1(2)}\right), \;\; \forall i \tag{10-2}$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\pi\right) y_{rj}^{t(1)} + \left(1-\pi\right) y_{rj}^{t(2)}\right) \ge \left(\left(1-\pi\right) y_{r0}^{t+1(2)} + \left(\pi\right) y_{r0}^{t+1(3)}\right), \;\; \forall r \tag{10-3}$$

$$\lambda_j \ge 0, \;\; \forall j \tag{10-4}$$

$$\Psi_0^{t+1}\left(x_0^{t}, y_0^{t}\right) = \text{Min} \;\; \theta \tag{11-1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\pi\right) x_{ij}^{t+1(2)} + \left(\pi\right) x_{ij}^{t+1(3)}\right) \le \theta\left(\left(\pi\right) x_{i0}^{t(1)} + \left(1-\pi\right) x_{i0}^{t(2)}\right), \;\; \forall i \tag{11-2}$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\pi\right) y_{rj}^{t+1(1)} + \left(1-\pi\right) y_{rj}^{t+1(2)}\right) \ge \left(\left(1-\pi\right) y_{r0}^{t(2)} + \left(\pi\right) y_{r0}^{t(3)}\right), \;\; \forall r \tag{11-3}$$

$$\lambda_j \ge 0, \;\; \forall j \tag{11-4}$$

Finally, the fuzzy Malmquist productivity index for the desired confidence level is calculated using Equation (12):

$$\text{FMPI}_0\left(\pi\right) = \sqrt{\frac{\Psi_0^{t}\left(x_0^{t+1}, y_0^{t+1}, \pi\right) * \Psi_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}, \pi\right)}{\Psi_0^{t}\left(x_0^{t}, y_0^{t}, \pi\right) * \Psi_0^{t+1}\left(x_0^{t}, y_0^{t}, \pi\right)}} \tag{12}$$

According to the value of the FMPI, which can be greater or equal to or less than one, the productivity change of the DMU under consideration for the desired confidence level is interpreted as follows:

- $\text{FMPI}_0(\delta) > 1$, the productivity increases and the progress is observed
- $\text{FMPI}_0(\delta) < 1$, the productivity decreases and the regress is observed
- $\text{FMPI}_0(\delta) = 1$, there is no change in productivity at time $t+1$ in comparison to t

5. Case study: Hospitals of Tehran

In this section, the applicability of FMPI that proposed in this research is implemented for measuring productivity changes of 10 hospitals in a real-life case study of Tehran. According to the experts' opinions, and literature review[2,3], the inputs and outputs are selected. The number of beds, the number of doctors, equipment & infrastructures and hospital location are considered as input variables. Also, the number of inpatient days, the number of outpatient, and overall patient satisfaction are selected as output variables. Data set of 10 hospitals for years 2013 and 2014 are presented in Tables (3) and (4), respectively:

Table (3): Hospitals Data for the Year 2013

Hospitals	Inputs				Outputs		
	Number of Beds	Number of Doctors	Equipment & Infrastructures	Hospital Location	Number of Inpatient Days	Number of Outpatient	Overall Patient Satisfaction
Hospital 01	535	540	VP	VP	139248	114004	VL
Hospital 02	187	126	G	F	31544	36984	L
Hospital 03	218	140	VG	VG	61622	177572	VH
Hospital 04	117	92	F	P	40594	35596	H
Hospital 05	121	93	G	F	24607	69580	M
Hospital 06	508	694	P	F	145773	150371	M
Hospital 07	69	179	F	G	11563	202089	VL
Hospital 08	340	164	F	VP	86034	96773	M
Hospital 09	114	148	G	G	27713	162846	L
Hospital 10	329	332	VP	F	96341	105928	L

Table (4): Hospitals Data for the Year 2014

Hospitals	Inputs				Outputs		
	Number of Beds	Number of Doctors	Equipment & Infrastructures	Hospital Location	Number of Inpatient Days	Number of Outpatient	Overall Patient Satisfaction
Hospital 01	521	404	P	VP	121352	104235	L
Hospital 02	188	109	G	F	38894	34544	M
Hospital 03	215	139	VG	VG	62076	157754	M
Hospital 04	118	83	G	P	40408	32893	VH
Hospital 05	110	84	F	F	23890	63236	M
Hospital 06	495	561	P	F	148280	147594	H
Hospital 07	69	123	G	G	12960	189377	M
Hospital 08	333	147	F	VP	83217	97272	L
Hospital 09	96	120	G	G	22900	151538	M
Hospital 10	320	151	VP	F	96971	94372	L

As previously discussed, patient satisfaction, equipment & infrastructures, and hospital location were measured with linguistic variables and the triangular fuzzy numbers (TFN) are presented in Table (5):

Table (5): The Linguistic Variables and Their Associated Triangular Fuzzy Numbers

Linguistic Variable	**Triangular Fuzzy Number**
Very Low (VL) / Very Poor (VP)	(0, 0, 0.25)
Low (L) / Poor (P)	(0, 0.25, 0.5)
Medium (M) / Fair (F)	(0.25, 0.5, 0.75)
High (H) / Good (G)	(0.5, 0.75, 1)
Very High (VH) / Very Good (VG)	(0.75, 1, 1)

Now, Models (8) to (11), are solved for different confidence levels, including 0%, 25%, 50%, 75, and 100%. The obtained results of Models (8) to (11), are presented in Tables (6) to (9), respectively:

Table (6): The Results of $\Psi_0^t\left(x_0^t, y_0^t, \pi\right)$

Hospitals	**Confidence Levels**				
	0%	25%	50%	75%	100%
Hospital 01	1.00000	1.26858	3.34180	3.50251	3.82363
Hospital 02	0.54471	0.54832	0.64787	1.05480	2.00000
Hospital 03	1.00000	1.10817	1.27047	1.51202	1.77778
Hospital 04	1.00000	1.77850	3.47881	3.98764	4.76519
Hospital 05	0.81306	0.96641	1.25158	1.74312	3.00000
Hospital 06	1.00000	1.89527	5.00000	6.14172	8.78816
Hospital 07	1.00000	1.32748	1.75958	2.35918	3.25325
Hospital 08	1.00000	1.37500	4.16667	6.19837	8.76265
Hospital 09	0.97211	1.04523	1.23889	1.45192	1.70926
Hospital 10	1.00000	3.39286	6.59152	7.10353	9.16615

Table (7): The Results of $\Psi_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right)$

Hospitals	**Confidence Levels**				
	0%	25%	50%	75%	100%
Hospital 01	1.00000	1.58723	2.05752	2.48914	2.94048
Hospital 02	0.64369	0.68248	0.96353	1.35745	2.01584
Hospital 03	1.00000	1.05896	1.14182	1.37310	1.76929
Hospital 04	1.00000	1.77778	3.42857	4.46719	8.61539
Hospital 05	0.87197	1.20283	1.67770	2.39276	3.69970
Hospital 06	1.00000	1.91803	4.20000	6.66667	9.00000
Hospital 07	1.00000	1.31049	1.71649	2.25333	2.97283
Hospital 08	1.00000	1.60133	3.25814	5.30917	7.83810
Hospital 09	0.95689	1.13296	1.47730	1.93554	2.54399
Hospital 10	1.00000	1.78252	4.56103	6.42857	9.76159

Table (8): The Results of $\Psi_0^t\left(x_0^{t+1}, y_0^{t+1}\right)$

Hospitals	Confidence Levels				
	0%	25%	50%	75%	100%
Hospital 01	1.00000	2.70074	3.32742	4.00373	6.14122
Hospital 02	0.75166	0.78187	1.00000	1.65753	3.00000
Hospital 03	1.00521	1.00846	1.02608	1.07032	1.33442
Hospital 04	1.47791	2.18439	3.96705	7.26492	9.60000
Hospital 05	0.86969	1.15598	1.69803	2.69236	4.00000
Hospital 06	1.33333	2.71429	4.00000	8.91699	9.95362
Hospital 07	1.95011	2.18033	2.45661	2.79428	3.21642
Hospital 08	1.12140	2.43936	4.24690	7.34954	9.88855
Hospital 09	1.30141	1.44756	1.63111	1.89458	2.17791
Hospital 10	2.21305	3.11473	4.99572	5.99338	8.00000

Table (9): The Results of $\Psi_0^{t+1}\left(x_0^t, y_0^t\right)$

Hospitals	Confidence Levels				
	0%	25%	50%	75%	100%
Hospital 01	1.28572	1.37660	1.40235	1.47835	1.51620
Hospital 02	0.52528	0.53791	0.64418	0.91302	1.47777
Hospital 03	1.21323	1.27648	1.47131	1.79217	2.20035
Hospital 04	1.07119	1.56545	3.01552	5.07692	7.46345
Hospital 05	0.76159	0.86920	1.16928	1.60705	2.36536
Hospital 06	0.99723	1.43255	3.01713	8.55556	9.87316
Hospital 07	1.51907	1.83637	2.25245	2.82196	3.69098
Hospital 08	1.98045	2.03435	3.44685	6.63254	8.84614
Hospital 09	0.95625	0.99918	1.17144	1.49649	1.98633
Hospital 10	1.12245	2.39742	5.00133	6.42857	8.00000

Finally, the results of fuzzy Malmquist productivity index are provided in Table (10) as follows:

Table (10): The Results of FMPI for Hospitals

Hospitals	Confidence Levels				
	0%	25%	50%	75%	100%
Hospital 01	0.88192	1.56675	1.20867	1.38733	1.76490
Hospital 02	1.30038	1.34505	1.51944	1.52851	1.43044
Hospital 03	0.91024	0.86888	0.79169	0.73644	0.77689
Hospital 04	1.17460	1.18102	1.13866	1.26612	1.52498
Hospital 05	1.10665	1.28658	1.39521	1.51649	1.44412
Hospital 06	1.15630	1.38473	1.05529	1.06364	1.01610
Hospital 07	1.13303	1.08264	1.03147	0.97250	0.89236
Hospital 08	0.75249	1.18172	0.98156	0.97424	0.99995
Hospital 09	1.15743	1.25314	1.28855	1.29912	1.27746
Hospital 10	1.40415	0.82618	0.83137	0.91854	1.03197

As can be seen in Table (10), five hospitals including Hospital 02, Hospital 04, Hospital 05, Hospital 06, and Hospital 09 have progressed under all confidence levels. Also, with respect to the FMPI results, the productivity of Hospital 03 is decreased in 2014 in comparison to 2013.

Finally, according to the results of the implementation of the presented FMPI in the real-life case study, the main advantages of the current research can be summarized as follows: The proposed FMPI can measure the productivity changes of hospitals over time in a fuzzy environment. The presented FMPI is capable of being applied in the presence of linguistic variables. In this regard, it should be noted the discriminatory power of FMPI is more than classic MPI. Moreover, the proposed models are linear, and as a result, by employing the common optimization software packages, the global optimal solution can be easily achieved.

6. Conclusions and Future Research Directions

One of the main applications of data envelopment analysis is to be integrated with the Malmquist productivity index in order to calculate the process of changes in productivity of DMU over different time periods.[38] The goal of this paper is to provide the Malmquist productivity index in order to calculate the productivity changes of hospitals with the fuzzy data. To achieve this goal, the possibilistic programming approach is applied. Moreover, for solving and showing validation of the proposed FMPI, a real-life case study of hospitals in Tehran was used. For future studies, the network DEA approach could be employed for performance assessment of hospitals by considering internal structure and relations.[39,40] Also, uncertain DEA models could be proposed based on robust optimization approach for performance measurement of hospitals under deep uncertainty.[41-45] Additionally, hybrid MCDM methods can be applied for performance assessment of hospitals.[46-48]

Abbreviations

MPI: Malmquist productivity index, **FMPI:** fuzzy Malmquist productivity index, **DEA:** data envelopment analysis, **DMU:** decision making unit, **CCR:** Charnes Cooper Rhodes, **BCC:** Banker Charnes Cooper, **SBM:** slack based measure, **SDEA:** stochastic data envelopment analysis, **FDEA:** fuzzy data envelopment analysis, **RDEA:** robust data envelopment analysis, **BDEA:** bootstrap data envelopment analysis, **SDEA:** interval data envelopment analysis, **MCDM:** multiple criteria decision making.

Competing Interests

The authors declare no competing interests.

Authors' Contributions

Pejman Peykani: Investigation, Conceptualization, Modelling, Data Preparation, Coding, Writing. **Fatemeh Sadat Seyed Esmaeili:** Conceptualization, Programming and Analyzing the Data, Writing. **Mohsen Rostamy-Malkhalifeh:** Supervision, Revision and Editing. **Farhad Hosseinzadeh Lotfi:** Supervision, Interpretation of the Results.

References

1. Bahadori M, Izadi AR, Ghardashi F, Ravangard R, Hosseini SM. The evaluation of hospital performance in Iran: a systematic review article. Iranian Journal of Public Health. 2016;45(7):855.
2. Kohl S, Schoenfelder J, Fügener A, Brunner JO. The use of data envelopment analysis (DEA) in healthcare with a focus on hospitals. Health Care Management Science. 2019;22(2):245-86.
3. Mahdiyan S, Dehghani A, Tafti AD, Pakdaman M, Askari R. Hospitals' efficiency in Iran: A systematic review and meta-analysis. Journal of Education and Health Promotion. 2019;8(1):126.
4. Farrell MJ. The measurement of productive efficiency. Journal of the Royal Statistical Society. Series A (General). 1957;120(3):253-290.
5. Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European Journal of Operational Research. 1978;2(6):429-44.
6. Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science. 1984;30(9):1078-92.
7. Peykani P, Mohammadi E, Seyed Esmaeili FS. Stock evaluation under mixed uncertainties using robust DEA model. Journal of Quality Engineering and Production Optimization. 2019.
8. Peykani P, Mohammadi E, Jabbarzadeh A, Jandaghian A. Utilizing robust data envelopment analysis model for measuring efficiency of stock, a case study: Tehran Stock Exchange. Journal of New Research in Mathematics. 2016;1(4):15-24.

9. Hatam N, Moslehi S, Askarian M, Shokrpour N, Keshtkaran A, Abbasi M. The efficiency of general public hospitals in Fars Province, Southern Iran. Iranian Red Crescent Medical Journal. 2010;12(2):138.
10. Lotfi F, Kalhor R, Bastani P, Zadeh NS, Eslamian M, Dehghani MR, Kiaee MZ. Various indicators for the assessment of hospitals' performance status: differences and similarities. Iranian Red Crescent Medical Journal. 2014;16(4).
11. Torabipour A, Najarzadeh M, Mohammad AR, Farzianpour F, Ghasemzadeh R. Hospitals productivity measurement using data envelopment analysis technique. Iranian Journal of Public Health. 2014;43(11):1576.
12. Raei B, Yousefi M, Rahmani K, Afshari S, Ameri H. Patterns of productivity changes in hospitals by using Malmquist–DEA Index: a panel data analysis (2011–2016). Australasian Medical Journal. 2017;10(10):856-64.
13. Alinezhad A. Malmquist productivity index using two-stage DEA model in heart hospitals. Iranian Journal of Optimization. 2018;10(2):81-92.
14. Ebrahimnejad A. Cost efficiency measures with trapezoidal fuzzy numbers in data envelopment analysis based on ranking functions: application in insurance organization and hospital. International Journal of Fuzzy System Applications. 2012;2(3):51-68.
15. Hatami-Marbini A, Tavana M, Emrouznejad A. Productivity growth and efficiency measurements in fuzzy environments with an application to health care. International Journal of Fuzzy System Applications. 2012;2(2):1-35.
16. Khaki AR, Sadjadi SJ, Gharakhani M, Rashidi S. Data envelopment analysis under uncertainty: A case study from public healthcare. African Journal of Business Management. 2012;6(24):7096-105.
17. Costantino N, Dotoli M, Epicoco N, Falagario M, Sciancalepore F. Using cross-efficiency fuzzy Data Envelopment Analysis for healthcare facilities performance evaluation under uncertainty. IEEE International Conference on Systems, Man, and Cybernetics. 2013;912-917.
18. De Nicola A, Gitto S, Mancuso P. Evaluating Italian public hospital efficiency using bootstrap DEA and CART. International Journal of Applied Decision Sciences. 2013;6(3):281-92.
19. Kalantary Z, Azar A. A robust data envelopment analysis model for ranking: A case of hospitals of Tehran. Data Envelopment Analysis and Performance Measurement. 2014;Proceedings of the 11th International Conference of DEA:21-28.

20. Karadayi MA, Karsak EE. Imprecise DEA framework for evaluating the efficiency of state hospitals in Istanbul. Proceedings of the World Congress on Engineering. 2014.
21. Haji-Sami E, Ebrahimi Khameneh M, Rezaeisaray M, Esmaeili S. Robust DEA under discrete uncertain data: An application for Iranian hospital emergency. 8th International Conference of Iranian Operations Research Society, Iran. 2015.
22. Kheirollahi H, Matin BK, Mahboubi M, Alavijeh MM. Chance constrained input relaxation to congestion in stochastic DEA. An application to Iranian hospitals. Global Journal of Health Science. 2015;7(4):151.
23. Mitropoulos P, Talias MA, Mitropoulos I. Combining stochastic DEA with Bayesian analysis to obtain statistical properties of the efficiency scores: An application to Greek public hospitals. European Journal of Operational Research. 2015;243(1):302-11.
24. Rabbani M, Heidari N, Farrokhi Asl H. A bootstrap interval robust data envelopment analysis for estimate efficiency and ranking hospitals. Journal of Industrial Engineering and Management Studies. 2016;3(2):107-22.
25. Arya A, Yadav SP. A fuzzy dual SBM model with fuzzy weights: an application to the health sector. Proceedings of 6th International Conference on Soft Computing for Problem Solving, Singapore. 2017;230-238.
26. Karsak EE, Karadayi MA. Imprecise DEA framework for evaluating health-care performance of districts. Kybernetes. 2017;46(4):706-27.
27. Kheirollahi H, Hessari P, Charles V, Chawshini R. An input relaxation model for evaluating congestion in fuzzy DEA. Croatian Operational Research Review. 2017;8(2):391-408.
28. Wu D, Wu DD. Risk-Based Robust Evaluation of Hospital Efficiency. IEEE Systems Journal. 2018;13(2):1906-14.
29. Hatefi SM, Haeri A. Evaluating hospital performance using an integrated balanced scorecard and fuzzy data envelopment analysis. Journal of Health Management and Informatics. 2019;6(2):66-76.
30. Peykani P, Mohammadi E, Emrouznejad A, Pishvaee MS, Rostamy-Malkhalifeh M. Fuzzy Data Envelopment Analysis: An Adjustable Approach. Expert Systems with Applications. 2019;136:439-452.
31. Färe R, Grosskopf S. Malmquist productivity indexes and Fisher ideal indexes. The Economic Journal. 1992;102(410):158-60.

32. Emrouznejad A, Tavana M. Performance Measurement with Fuzzy Data Envelopment Analysis. Springer; 2014.
33. Hatami-Marbini A, Emrouznejad A, Tavana M. A taxonomy and review of the fuzzy data envelopment analysis literature: two decades in the making. European Journal of Operational Research. 2011;214(3):457-72.
34. Lertworasirikul S, Fang SC, Joines JA, Nuttle HL. Fuzzy data envelopment analysis (DEA): a possibility approach. Fuzzy Sets and Systems. 2003;139(2):379-94.
35. Peykani P, Mohammadi E, Pishvaee MS, Rostamy-Malkhalifeh M, Jabbarzadeh A. A novel fuzzy data envelopment analysis based on robust possibilistic programming: possibility, necessity and credibility-based approaches. RAIRO-Operations Research. 2018;52(4):1445-63.
36. Peykani P, Mohammadi E, Rostamy-Malkhalifeh M, Hosseinzadeh Lotfi F. Fuzzy data envelopment analysis approach for ranking of stocks with an application to Tehran stock exchange. Advances in Mathematical Finance and Applications. 2019;4(1):31-43.
37. Zadeh LA. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems. 1978;1(1):3-28.
38. Emrouznejad A, Yang GL. A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. Socio-Economic Planning Sciences. 2018;61:4-8.
39. Peykani P, Mohammadi E. Interval network data envelopment analysis model for classification of investment companies in the presence of uncertain data. Journal of Industrial and Systems Engineering. 2018;11(Special issue: 14th International Industrial Engineering Conference):63-72.
40. Peykani P, Mohammadi E. Window network data envelopment analysis: an application to investment companies. International Journal of Industrial Mathematics. 2019.
41. Hosseini-Motlagh SM, Ghatreh Samani MR, Cheraghi S. Robust and stable flexible blood supply chain network design under motivational initiatives. Socio-Economic Planning Sciences. 2019.
42. Hosseini-Motlagh SM, Ghatreh Samani MR, Homaei S. Blood supply chain management: robust optimization, disruption risk, and blood group compatibility (a real-life case). Journal of Ambient Intelligence and Humanized Computing. 2019:1-20.

43. Ghatreh Samani MR, Hosseini-Motlagh SM, Ghannadpour SF. A multilateral perspective towards blood network design in an uncertain environment: Methodology and implementation. Computers & Industrial Engineering. 2019;130:450-71.
44. Ghatreh Samani M, Hosseini-Motlagh SM. A hybrid algorithm for a two-echelon location-routing problem with simultaneous pickup and delivery under fuzzy demand. International Journal of Transportation Engineering. 2017;5(1):59-85.
45. Cheraghi S, Hosseini-Motlagh SM, Ghatreh Samani M. Integrated planning for blood platelet production: a robust optimization approach. Journal of Industrial and Systems Engineering. 2017;10(special issue on healthcare):55-80.
46. Sedady F, Beheshtinia MA. A novel MCDM model for prioritizing the renewable power plants' construction. Management of Environmental Quality: An International Journal. 2019;30(2):383-99.
47. Beheshtinia MA, Omidi S. A hybrid MCDM approach for performance evaluation in the banking industry. Kybernetes. 2017;46(8):1386-407.
48. Torkzad A, Beheshtinia MA. Evaluating and prioritizing hospital service quality. International Journal of Health Care Quality Assurance. 2019;32(2):332-46.