

# Efficiency Measurement of Clinical Units Using Integrated Independent Component Analysis-DEA Model under Fuzzy Conditions

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## Abstract

**Background and Objectives:** Evaluating the performance of clinical units is critical for effective management of health settings. Certain assessment of clinical variables for performance analysis is not always possible, calling for use of uncertainty theory. This study aimed to develop and evaluate an integrated independent component analysis-fuzzy-data envelopment analysis approach to accurately measure the performance of clinical units under uncertainty.

**Methods:** Correlations between the input variables were calculated using Pearson's correlation coefficient. Independent component analysis was used to extract independent components from input variables. Independent components were filtered against Gaussianity using Kurtosis parameter. An integrated independent component analysis-fuzzy-data envelopment analysis method was developed by using the uncertainty theory in the nonlinear fractional model proposed by Charnes, Cooper, and Rhodes (1978). The resulting fuzzy efficiency numbers were converted into normal ranking values by calculating a matrix of degree of preference.

**Findings:** Under certainty, while data envelopment analysis identified 12 out of the 19 units as efficient units, independent component analysis-data envelopment analysis approach identified only three efficient units. On the other hand, under fuzzy conditions, while fuzzy-data envelopment analysis identified 12 efficient units, independent component analysis-fuzzy-data envelopment analysis identified only three units as efficient units.

**Conclusions:** The results indicated that independent component analysis-fuzzy-data envelopment analysis offers the same efficiency measurement performance under fuzzy conditions as corresponding non-fuzzy method does under certain conditions. Our findings, hence, recommend use of the new approach in estimating efficiency of clinical units when access to reliable data is limited.

**Keywords:** Hospital Performance, Data Envelopment Analysis, Independent Component Analysis, Fuzzy Theory, Clinical Units

## Background and Objectives

Quality of health services provided and the efficiency of the healthcare related operations are among the major concerns of health system policy-makers in countries with large public healthcare budget [1]. Valid evaluation and ranking of clinical DMUs is critical for efficient management of health settings and improving their performance. Among the several efficiency measurement approaches such as the conventional statistical methods and the non-parametric methods, Data

Envelopment Analysis (DEA) has proven to be more effective in measuring the relative efficiencies of multiple Decision-Making Units (DMUs).

DEA's major advantage for use in healthcare domain is related to its flexibility and versatility; this method requires no information on relative costs, and can easily handle multiple inputs and outputs. However, these strengths also give rise to some practical limitations. For instance, if the inputs of a DMU are strongly correlated, the efficiency estimation of this DMU may become invalid [2]. To address this issue, a number of strategies have been proposed. Vitner *et al.* [2] introduced a similarity coefficient algorithm that uses a grouping process to reduce the number of inputs and outputs while maintaining the essential

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information. Adler and Yazhensky [3] suggested the use of Principal Component Analysis (PCA) to generate uncorrelated original inputs and outputs. Kao *et al.* [1] proposed the use of Independent Component Analysis (ICA) to extract the input variables by identifying the independent components.

DEA has been widely used independently or in combination with other methods in different industries, including energy, banks, airlines, and hospital industries (Table 1). However, as seen in Table 1, the usefulness of combining ICA, DEA, and Fuzzy Logic has not been examined yet. In this study, we propose a hybrid Independent Component Analysis–fuzzy Data Envelopment Analysis (ICA–fDEA) approach for evaluating the performance of clinical units in health settings.

In real-world problems, values of the input and output data may not be precisely determined. Indeed, in reality, performance variables could be uncertain, calling for use of fuzzy approaches to performance measurement. By using an ICA–fDEA to clinical data, this study attempts to examine the performance of this measurement method in evaluating the performance of clinical units.

## Methods

Figure 1 illustrates the stages of ICA–fDEA approach to clinical performance evaluation. First, the input and output variables are determined. Correlation analysis identifies the correlated input variables. If the correlations are significant, the input data are converted into separate independent data using ICA. In the next step, the separated data are used in fDEA to yield fuzzy performance values. The fuzzy performance values are converted into normal values by calculating the matrix of degree of preference.

### Definition of Inputs and Outputs

The validity of performance measurement is largely related to accurate selection of the input and output variables. Therefore, an extensive literature review was conducted for selection of the study variables [4–9, 5, 6, 7, 8, 9]. Five input and two output variables were chosen to calculate the efficiency of hospital DMUs (*i.e.*, clinical units). The input variables included number of beds, number of doctors, number of nurses, cost of equipment, and number of supporting medical staff (ancillary personnel). On the other hand, the frequency of inpatient visits, and the rate of bed occupancy were considered as output variables.

### Calculation of correlations in the input and output data

As mentioned earlier, once the input variables are strongly correlated, the efficiency estimates of DMU obtained from the DEA can become invalid; hence, it is required to calculate the correlations between each pair of input-output.

### Independent Component Analysis (ICA)

High correlation between variables can significantly affect the measurement of efficiency. Therefore, there is a need to convert the observed input data into separate independent signals by the ICA approach before conducting DEA. Independent Component Analysis (ICA) is a powerful statistical technique that allows the transformation of the observed correlated signals into statistically independent signals. The linear ICA model is formulated according to Equation 1:

$$\mathbf{x}(t_i) = A \cdot \mathbf{s}(t_i) \quad (1)$$

where  $\mathbf{s}(t_i) = [s_1(t_i), \dots, s_N(t_i)]$  is the  $N$  dimensional vector of unknown source signals,  $\mathbf{x}(t_i) = [x_1(t_i), \dots, x_M(t_i)]$  is the  $M$  dimensional vector of observed signals, and  $A$  is an  $M \times N$  matrix called mixing matrix. The number of observed signals is usually greater than or equal to the number of sources ( $M \geq N$ ) [10]. Let  $\mathbf{x}$  denotes the random vector whose elements are a mixture of  $x_1, \dots, x_M$ , and  $\mathbf{s}$  denotes the random vector with elements  $s_1, \dots, s_N$ . Let  $A$  indicates the matrix with elements  $a_{ij}$ . The mixture model using the vector-matrix is defined as follows:

$$\mathbf{x} = A \cdot \mathbf{s} \quad ; \quad \text{or} \quad \begin{bmatrix} x_1 \\ \bullet \\ \bullet \\ \bullet \\ x_M \end{bmatrix} = \begin{bmatrix} a_{11} & \bullet & \bullet & a_{1N} \\ \bullet & \bullet & \bullet & \bullet \\ a_{M1} & \bullet & \bullet & a_{MN} \end{bmatrix} \begin{bmatrix} s_1 \\ \bullet \\ \bullet \\ \bullet \\ s_N \end{bmatrix} \quad (2)$$

The ICA model is a generative model because it describes how the observed data are generated by the process of mixing the components  $s_j$ . The only known data in Equation 2 is the vector  $\mathbf{x}$ ; hence, both matrix  $A$  and vector  $\mathbf{s}$  should be estimated. The basic assumptions for using ICA are as follows:

- i) The components  $s_j$  (*i.e.*, the sources) are statistically independent as much as possible.
- ii) The independent components must follow a non-Gaussian distribution (at most, one source can have Gaussian distribution).

**Table1 Taxonomy of DEA applications**

| Application           | DEA | FUZZY DEA | ICA-DEA | Reference |
|-----------------------|-----|-----------|---------|-----------|
| Energy                | ✓   |           |         | [7]       |
|                       | ✓   |           |         | [8]       |
|                       |     | ✓         |         | [9]       |
| Project               | ✓   |           |         | [10]      |
| Bank                  |     | ✓         |         | [11]      |
| Hospital              |     |           | ✓       | [1]       |
|                       | ✓   |           |         | [12]      |
|                       | ✓   | ✓         |         | [13]      |
|                       | ✓   |           |         | [14]      |
|                       | ✓   |           |         | [15]      |
| University            |     | ✓         |         | [16]      |
|                       | ✓   |           |         | [17]      |
| Manufacturing systems |     | ✓         |         | [18]      |
|                       |     | ✓         |         | [19]      |
| Airline               | ✓   |           |         | [20]      |
|                       | ✓   |           |         | [21]      |
| Business              |     | ✓         |         | [22]      |

The ICA model aims at finding an  $mm$  de-mixing matrix  $W$  so that  $Y = W.S$ ; where  $Y(y_1, y_2, \dots, y_n)$  (the matrix of independent components (ICs)) is statistically independent. The FastICA algorithm developed by Hyvärinen *et al.* [10] was adopted to determine the de-mixing matrix  $W$ .

As mentioned before, the data should be non-Gaussian for use in ICA. Hence, a quantitative evaluation of non-Gaussianity is required. We examined non-Gaussianity using Kurtosis parameter. Kurtosis describes the shape of the probability distribution function. Since Gaussian distribution has a normalized Kurtosis of zero, a non-zero kurtosis value can indicate non-Gaussianity [10]. If the *expected* value of random variable  $y$  is denoted by  $E\{y\}$ , the Kurtosis of variable  $y$ , can be defined as:

**Fuzzy DEA**

Charnes, Cooper, and Rhodes (CCR) (1978) designed a

$$kurt(y) = \frac{E\{y^4\}}{E\{y^2\}^2} - 3 \tag{3}$$

nonlinear fractional DEA model, known as the CCR model [4]. Let each DMU has  $m$  inputs ( $i = 1, \dots, m$ ) and  $s$  outputs ( $r = 1, \dots, s$ ). The CCR model is then formulated as follows [11]:

$$\theta_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \tag{4}$$

where  $u_r$  and  $v_i$  are the weights given to output  $r$  and input  $i$ , respectively.

Input variables (in this study, the number of beds, doctors, nurses and cost of equipment) may not be precisely determined in certain situations. Such data should, therefore, be estimated using fuzzy numbers. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. The most commonly used fuzzy numbers are triangular fuzzy numbers. The *membership functions* are defined as below:

$$\mu(x) = \begin{cases} (x - a)/(b - a) & a \leq x \leq b \\ (d - x)/(d - b) & b \leq x \leq d \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Where,  $a$ ,  $b$ , and  $d$  represent triangular fuzzy numbers. Suppose there are  $n$  DMUs. Each  $DMU_j (j = 1, \dots, n)$  uses a set of fuzzy inputs  $x_{ij} (i = 1, \dots, m)$  to produce a set of fuzzy outputs  $\bar{y}_{rj} (r = 1, \dots, s)$ ; where,  $x_{ij}$  and  $\bar{y}_{rj}$  are fuzzy numbers. Then, the fuzzy CCR model can be formulated as follows:

**Table 2 Selection of input and output variables**

|        | Variables           | Mean    | Min   | Max   |
|--------|---------------------|---------|-------|-------|
| Input  | Beds                | 19.73   | 8     | 36    |
|        | Doctors             | 5.84    | 0     | 19    |
|        | Nurses              | 12.73   | 6     | 23    |
|        | Cost of equipment   | 303.42  | 60    | 950   |
|        | Ancillary personnel | 1.57    | 1     | 3     |
| Output | Inpatient visits    | 1429.84 | 365   | 4053  |
|        | Bed occupancy       | 74.01   | 33.25 | 94.17 |

$$\tilde{\theta}_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \tag{6}$$

where  $\theta$  is a fuzzy number referred to as a fuzzy efficiency,  $u_r (r = 1, \dots, s)$  and  $v_i (i = 1, \dots, m)$  are the weights assigned to the inputs and outputs, respectively. Based on the basic principles of fuzzy mathematics, the fuzzy efficiency defined in Equation 6 can be expressed as below [12]:

$$\tilde{\theta}_j = \frac{\sum_{r=1}^s u_r [y_{rj}^l, y_{rj}^M, y_{rj}^U]}{\sum_{i=1}^m v_i [x_{ij}^l, x_{ij}^M, x_{ij}^U]} \tag{7}$$

$$\tag{8}$$

$$\tilde{\theta}_j \approx \left[ \frac{\sum_{r=1}^s u_r y_{rj}^l}{\sum_{i=1}^m v_i x_{ij}^U}, \frac{\sum_{r=1}^s u_r y_{rj}^M}{\sum_{i=1}^m v_i x_{ij}^M}, \frac{\sum_{r=1}^s u_r y_{rj}^U}{\sum_{i=1}^m v_i x_{ij}^l} \right]$$

Let  $\tilde{a} = (a_p, a_m, a_u)$  and  $b = (b_p, b_m, b_u)$  represent triangular fuzzy efficiencies. Then the degree of preference of  $\tilde{a} > b$  is defined as [12]:

$$P(a > b) = \begin{cases} 1 & \text{if } a_l \geq b_u \\ 0 & \text{if } a_u \leq b_l \\ \frac{(a_u - b_l)^2}{(a_u - b_l + b_m - a_m)(a_u - a_l + b_u - b_l)} & \text{if } (a_u > b_l) \cap (a_m \leq b_m) \\ 1 - \frac{(b_u - a_l)^2}{((b_u - a_l + a_m - a_m)(a_u - a_l + b_u - b_l))} & \text{if } (a_m > b_m) \cap (a_l < b_u) \end{cases} \tag{9}$$

Fuzzy DEA results in a fuzzy efficiency score. Identifying the relative efficiency of DMU from the fuzzy efficiency scores is difficult. Hence, there is a need for conversion of fuzzy efficiency scores into conventional ranking numbers. There are several methods for ranking the fuzzy numbers. This study adopts the approach proposed by Wang *et al.* [12] to rank the fuzzy numbers, which is based on the matrix of degree of preference ( $M_p$ ):

$$M_p = \begin{bmatrix} - & P_{12} & \dots & P_{1n} \\ P_{21} & - & \dots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \dots & - \end{bmatrix} \tag{10}$$

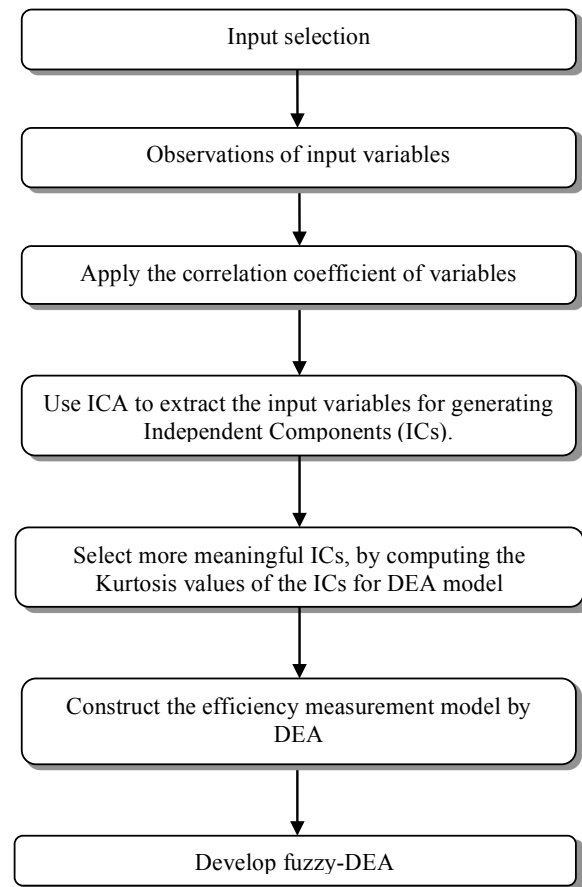
where  $P_{ij} = P(\theta_i > \theta_j)$  is determined by Equation 9. Finally a row from with all elements larger than or equal to 0.5 is found. If this row corresponds to  $\theta_p$ , then  $\theta_p$  is the largest fuzzy efficiency based on which the  $i$ th row and  $j$ th column will be removed from the matrix, and the process is repeated until all fuzzy efficiencies are properly ranked.

**Table 4 Kurtosis value of ICs**

| IC1  | IC2  | IC3  | IC4  | IC5  |
|------|------|------|------|------|
| 4.66 | 1.77 | 5.77 | 6.62 | 1.59 |

**Table 5 CCR and ICA-CCR scores and their ranks**

| DMU | CCR Score | ICA-CCR score |
|-----|-----------|---------------|
| 1   | 0.79      | 0.68          |
| 2   | 1         | 0.90          |
| 3   | 1         | 1             |
| 4   | 1         | 0.77          |
| 5   | 1         | 0.96          |
| 6   | 1         | 0.54          |
| 7   | 1         | 0.68          |
| 8   | 1         | 0.87          |
| 9   | 0.61      | 0.31          |
| 10  | 0.89      | 0.73          |
| 11  | 0.98      | 0.87          |
| 12  | 0.63      | 1             |
| 13  | 1         | 0.48          |
| 14  | 1         | 0.44          |
| 15  | 1         | 0.80          |
| 16  | 1         | 0.45          |
| 17  | 1         | 0.94          |
| 18  | 1         | 0.77          |
| 19  | 1         | 1             |



**Figure 1** Stages of ICA-fDEA model

**Table 3 Correlations among variables**

|                     | Beds    | Doctors | Ancillary personnel | Nurses | Cost of equipment | Bed occupancy | Inpatient visits |
|---------------------|---------|---------|---------------------|--------|-------------------|---------------|------------------|
| Beds                | 1       |         |                     |        |                   |               |                  |
| Doctors             | 0.11396 | 1       |                     |        |                   |               |                  |
| Ancillary personnel | -0.26   | -0.13   | 1                   |        |                   |               |                  |
| Nurses              | 0.96    | 0.20    | -0.19               | 1      |                   |               |                  |
| Cost of equipment   | -0.02   | -0.20   | 0.31                | 0.025  | 1                 |               |                  |
| Bed occupancy       | 0.074   | -0.42   | -0.044              | 0.07   | 0.10              | 1             |                  |
| Inpatient visits    | 0.80    | 0.22    | 0.03                | 0.87   | 0.04              | -0.08         | 1                |

**Table 6** Summary of the results of standard DEA and ICA–DEA models

|                            | CCR  | ICA-CCR |
|----------------------------|------|---------|
| Number of DMUs             | 19   | 19      |
| Average score              | 0.94 | 0.74    |
| Standard deviation         | 0.12 | 0.21    |
| Maximum score              | 1    | 1       |
| Minimum score              | 0.61 | 0.31    |
| Number of efficient DMUs   | 14   | 3       |
| Number of inefficient DMUs | 5    | 16      |

**Table 7** Scores of fCCR and ICA-fCCR Algorithms

| DMU | fCCR             | ICA-fCCR         |
|-----|------------------|------------------|
| 1   | (0.78,0.79,0.80) | (0.65,0.68,0.70) |
| 2   | (1,1,1)          | (0.87,0.89,0.94) |
| 3   | (1,1,1)          | (1,1,1)          |
| 4   | (1,1,1)          | (0.71,0.77,0.83) |
| 5   | (1,1,1)          | (0.92,0.96,1)    |
| 6   | (1,1,1)          | (0.44,0.51,0.53) |
| 7   | (1,1,1)          | (0.43,0.67,0.69) |
| 8   | (1,1,1)          | (0.84,0.85,0.87) |
| 9   | (0.60,0.61,0.62) | (0.23,0.31,0.32) |
| 10  | (0.89,0.90,0.93) | (0.56,0.67,0.72) |
| 11  | (0.94,0.98,1)    | (0.82,0.85,0.87) |
| 12  | (0.58,0.63,0.70) | (1,1,1)          |
| 13  | (0.98,1,1)       | (0.38,0.46,0.48) |
| 14  | (0.93,0.98,1)    | (0.33,0.41,0.43) |
| 15  | (1,1,1)          | (0.56,0.70,0.79) |
| 16  | (1,1,1)          | (0.43,0.45,0.46) |
| 17  | (1,1,1)          | (0.67,0.85,0.94) |
| 18  | (1,1,1)          | (0.62,0.77,0.78) |
| 19  | (1,1,1)          | (1,1,1)          |

## Results and Discussion

### Correlation Analysis

The results of correlation analysis are reported in Table 3. All correlations are significant at 0.01 level. The small-

est correlation coefficient is related to the relationship between the number of nurses and the cost of equipment. On the other hand, the largest correlation coefficient is seen in the relationship between the number of nurses and the number of beds.

**Table 8 Degree of preference for ICA-fCCR efficiencies and their ranks**

|    | 1      | 2      | 3        | 4      | 5 | 6      | 7      | 8     | 9        | 10     | 11 | 12     | 13    | 14     | 15     | 16     | 17     | 18     | 19 | Rank |
|----|--------|--------|----------|--------|---|--------|--------|-------|----------|--------|----|--------|-------|--------|--------|--------|--------|--------|----|------|
| 1  | -      | 0      | 0.0      | 0      | 0 | 1      | 0.8967 | 0     | 1.0.9365 | 0      | 0  | 1      | 1     | 1      | 0.0445 | 1      | 0.014  | 0.1792 | 0  | 12   |
| 2  | 1      | -      | 0.1      | 0.0296 | 1 | 1      | 1      | 1     | 1.1      | 1      | 0  | 1      | 1     | 1      | 1      | 1      | 0.8689 | 1      | 0  | 5    |
| 3  | 1      | 1      | -        | 1      | 1 | 1      | 1      | 1     | 1.1      | 1      | 1  | 1      | 1     | 1      | 1      | 1      | 1      | 1      | 1  | 1    |
| 4  | 1      | 1      | 0        | 0      | 0 | 1      | 1      | 0     | 1.0.9967 | 0.0065 | 0  | 1      | 1     | 1      | 0.878  | 1      | 0.2853 | 0.75   | 0  | 9    |
| 5  | 1      | 0.9704 | 0.1      | -      | 1 | 1      | 1      | 1     | 1.1      | 1      | 1  | 0      | 1     | 1      | 1      | 1      | 0.9912 | 1      | 0  | 4    |
| 6  | 0      | 0      | 0.0      | 0      | 0 | -      | 0.1098 | 0     | 1.0      | 0      | 0  | 0.9064 | 1     | 0      | 0.9583 | 0      | 0      | 0      | 0  | 15   |
| 7  | 0.1033 | 0      | 0.0      | 0      | 0 | 0.8902 | -      | 0     | 1.0.3095 | 0      | 0  | 0.974  | 1     | 0.2155 | 0.9793 | 0.0039 | 0.0686 | 0      | 0  | 14   |
| 8  | 1      | 0      | 0.1      | 0      | 0 | 1      | 1      | -     | 1.1      | 0.625  | 0  | 1      | 1     | 1      | 1      | 1      | 0.6666 | 1      | 0  | 6    |
| 9  | 0      | 0      | 0.0      | 0      | 0 | 0      | 0      | 0     | -        | 0      | 0  | 0      | 0     | 0      | 0      | 0      | 0      | 0      | 0  | 19   |
| 10 | 0.635  | 0      | 0.0.0033 | 0      | 1 | 0.6905 | 0      | 1     | 1        | 0      | 0  | 1      | 1     | 0.3454 | 1      | 0.0002 | 0.1562 | 0      | 0  | 13   |
| 11 | 1      | 0      | 0.0.9935 | 0      | 1 | 1      | 1      | 0.375 | 1.1      | -      | 0  | 1      | 1     | 1      | 1      | 1      | 0.0625 | 1      | 0  | 8    |
| 12 | 1      | 1      | 1.1      | 1      | 1 | 1      | 1      | 1     | 1.1      | 1      | 1  | -      | 1     | 1      | 1      | 1      | 1      | 1      | 1  | 1    |
| 13 | 0      | 0      | 0.0      | 0      | 0 | 0.0936 | 0.026  | 0     | 1.0      | 0      | 0  | -      | 0.875 | 0      | 0.4529 | 0      | 0      | 0      | 0  | 17   |
| 14 | 0      | 0      | 0.0      | 0      | 0 | 0      | 0      | 0     | 1.0      | 0      | 0  | 0.125  | -     | 0      | 0      | 0      | 0      | 0      | 0  | 18   |
| 15 | 0.9555 | 0      | 0.0.122  | 0      | 1 | 0.7845 | 0      | 1     | 1.0.6546 | 0      | 0  | 1      | 1     | -      | 1      | 0.1066 | 0.3087 | 0      | 0  | 11   |
| 16 | 0      | 0      | 0.0      | 0      | 0 | 0.0417 | 0.0207 | 0     | 1.0      | 0      | 0  | 0.5471 | 1     | 0      | -      | 0      | 0      | 0      | 0  | 16   |
| 17 | 0.986  | 0.1311 | 0.0.7147 | 0.0088 | 1 | 0.9961 | 0.3334 | 1     | 1.0.9998 | 0.9375 | 0  | 1      | 1     | 0.8934 | 1      | -      | 0.8518 | 0      | 0  | 7    |
| 18 | 0.8208 | 0      | 0.0.25   | 0      | 1 | 0.9314 | 0      | 1     | 1.0.8438 | 0      | 0  | 1      | 1     | 0.6913 | 1      | 0.1482 | -      | 0      | 0  | 10   |
| 19 | 1      | 1      | 1.1      | 1      | 1 | 1      | 1      | 1     | 1.1      | 1      | 1  | 1      | 1     | 1      | 1      | 1      | 1      | 1      | -  | 1    |

## ICA-DEA

In the ICA-DEA procedure, first ICA was applied to estimate decomposition matrix  $W$ , thereby determining independent components. In order to select more significant ICs, the statistical independence of ICs was evaluated by calculating their Kurtosis [1]. The estimated Kurtosis values for each IC are summarized in Table 4. ICs with kurtosis larger than three were considered as the most significant ones [10]; thereby, IC1, IC3 and IC4 are the key factors affecting efficiency measurement. These ICs, therefore, were used as three new input variables for the DEA model.

Once the ICs were determined, the efficiency scores were computed by the CCR input-oriented model, proposed by Charnes *et al.* [11] and ICA-CCR. Table 5 compares the results of both CCR and ICA-CCR models.

The overall efficiency scores of 19 hospital units (DMUs) are summarized in Table 6. As seen, the traditional DEA model (CCR model) has estimated a high average efficiency score (0.94) with a small standard deviation (0.12). Hence, it has failed to make a significant distinction among the DMUs by identifying 14 out of 19 DMUs to be completely efficient [1]. However, the ICA-CCR model has identified only three units out of 19 as efficient units, which clearly outperforms the CCR method.

## ICA-fDEA

Assuming that the number of beds, doctors, nurses and cost of equipment are not precise, these data were, therefore, estimated using fuzzy triangular numbers, followed by calculation of fuzzy efficiency scores for each hospital units using Equation 8. Table 7 compares the results of fDEA and ICA-fDEA in 19 clinical units. As seen, while fDEA has identified 12 DMUs as efficient DMUs, ICA-fDEA has identified only three efficient DMUs, which indicate that ICA-fDEA offers the same performance for fuzzy data as ICA-DEA for certain data.

## Ranking fuzzy efficiency score

The fuzzy efficiencies reported in Table 7 cannot be directly used for ranking DMUs. Therefore, for ranking performance of the 19 clinical units in ICA-fDEA, first, the matrix of degree of preference ( $\rho$ ) was calculated. Table 8 shows the matrix calculated using Equation 10 for ranking the fuzzy efficiencies.

## Conclusions

In this study, an independent component analysis-fuzzy-data envelopment analysis (ICA-fDEA) approach was developed for improving the discriminatory capability of traditional DEA in estimating the efficiency of decision-making units when inputs are both correlated and fuzzy. The developed method was shown to be able to identify efficient decision-making units under fuzzy conditions with the same performance as does ICA-fDEA model in certain conditions. Our results, hence, recommend use of ICA-fDEA for estimating the efficiency of clinical units when access to accurate data is limited.

## Abbreviations

(FDEA): Fuzzy data envelopment analysis; (ICA): Independent component analysis.

## Competing Interests

The authors declare no competing interests.

## Authors' Contributions

AA, MB and KF jointly designed the study. AA contributed to data collection and analysis, interpretation of results, and editing the draft manuscript. MB was involved in editing the draft manuscript. AA, MB and KF contributed to revising the manuscript. All authors read and approved the final manuscript.

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