

# Competitive Location Model in Healthcare: A Case Study on Tehran's Health Centers

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## Abstract

**Background and Objectives:** The location of facilities is of great importance in healthcare and is of interest to researchers. In this regard, a large proportion of classic location-allocation models concentrate on solving problems in an exclusive environment (non-competitive), but this assumption is rarely true in reality.

**Methods:** At first, a basic Non-Competitive Location Model (NCLM) is presented. Then, a Competitive Location Model (CLM) is developed based on the initial model. This study proposes a multi-objective integer programming model based on Nash bargaining game. The first objective function maximizes the two-person Nash product, which in turn maximizes the total number of patients covered by the newly established healthcare centers. The second objective function minimizes the sum of distances between population centers and the newly established healthcare centers.

**Findings:** The results obtained from applying the CLM on Tehran's Health Centers revealed the capabilities of this model in simulating the competitive situations. For instance, by comparing the results obtained from both models (non-competitive and competitive) it became clear that the total covered population was considerably increased in the CLM.

**Conclusions:** The proposed location model can be used as a basis for decision making by managers. Because of any wrong decision, in addition to raising the costs of the health system, can also lead to irreparable damage to human and social health.

**Keywords:** Health centers location-allocation, Bargaining game, Multi-objective programming, Competitive environment, Healthcare in metropolis

## Background and objectives

In healthcare service sector, poor location decisions may lead to inappropriate consequences such as increase in mortality and morbidity. Thus, facility location modeling is highly important whenever sitting in healthcare facilities is considered.<sup>1</sup> In other words, the design of the health service network is one of the most significant strategic decisions that affect health systems efficiency to a high extent.<sup>2</sup> Furthermore, in the National Health Service (NHS), changing the healthcare network may be essential due to efficiency reasons such as reduction of operating costs, maintaining efficacy and inequality containment in the distribution of accessibility costs.<sup>3,4</sup> In this case, before establishing a new health center in a certain area, several factors must be taken into account. Unfortunately, this principle is often neglected in developing countries and

new health centers are established and equipped without considering a comprehensive investigation.

A health center, whether serving an urban or rural population, is an integral part of the country's National Health Service,<sup>5</sup> with the responsibility for offering care with high quality for its members.<sup>6,7</sup> In this regard, location-allocation models have played a significant role in making healthcare centers more accessible to patients.<sup>8</sup> Therefore, there is a need for mathematical formulations that simplify the management of health services. However, investigations conducted in the area of healthcare are abundant and drawn researchers attention during recent years. Daskin and Dean<sup>1</sup> derived three basic facility location models form the core of healthcare location problems and most of other models are adopted from these three models. These models include the set covering model, the maximal covering model, and the p-median model. Other research in this area include Daskin,<sup>9</sup> Francis et al<sup>10</sup> and Handler and Mirchandani.<sup>11</sup> Recently, Berman et al<sup>12</sup> provided a review

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of covering location models and outlined the most recent developments in this area. Similarly, Zanjirani Farahani et al<sup>13</sup> presented a comprehensive review of covering location problems and outlined the models, solutions, and applications related to this area of interest. Daskin and Dean<sup>1</sup> categorized the related literature into three broad areas: accessibility models, adaptability models, and availability models.

Marianov and ReVelle<sup>14</sup> reviewed emergency facility location models. A paper on emergency facility location under network damage is given by Salman and Yücel.<sup>15</sup> Rahman and Smith<sup>16</sup> studied the application of location-allocation models in health service planning in developing countries and demonstrated the usefulness of location-allocation models in such countries. Asefzadeh<sup>17</sup> assessed the necessity of establishing new hospitals and states that numerous factors must be considered before establishing a new healthcare center in a specific area. Goldstein et al<sup>18</sup> investigated the effect of some important factors on hospital performance such as location, strategy, and operations technology. In the following, integrated geographical information system-fuzzy analytical hierarchy process,<sup>19</sup> mixed-integer programming,<sup>20</sup> binary integer programming,<sup>21</sup> data envelopment analysis,<sup>3</sup> mixed integer non-linear programming,<sup>22</sup> bi-objective mathematical programming,<sup>23</sup> Stochastic planning,<sup>24</sup> optimization approaches,<sup>25</sup> mixed-integer non-linear programming,<sup>26</sup> mixed possibilistic-flexible programming,<sup>27</sup> network information system<sup>28</sup> were used to design of healthcare network and locate the health centers. Meanwhile, other studies have been done on the blood supply chain network design. Among the approaches used in these studies can be noted fuzzy analytic hierarchy process and grey rational analysis,<sup>29</sup> mixed-integer linear programming,<sup>30-32</sup> multi-objective mixed-integer linear programming,<sup>33,34</sup> robust possibilistic programming,<sup>35</sup> and two-stage stochastic programming.<sup>36</sup>

A company is rarely the only player in the market, and it is more realistic to assume that it must compete with other firms. Whenever the location of a new facility is investigated, many factors come to play. An essential factor is whether other competitors exist in the market or not, Hotelling<sup>37</sup> is a pioneer in the field of competitive facility location. A review on this type of location problems can be seen in different papers such as Plastria,<sup>25</sup> Eiselt and Laporte,<sup>38</sup> Drezner<sup>39</sup> and Eiselt et al.<sup>40</sup> Competitive facility location models differ by changing the elements that form these models. These are the type of competition, features of the market and decision space.<sup>25</sup> Oostrum et al<sup>41</sup> considered a competitive facility location problem in their paper and investigated the Voronoi game. Ashtiani et al<sup>42</sup> presented

a robust leader-follower model for locating facilities in a discrete space. Drezner et al<sup>43</sup> proposed a new approach for estimating market share when modeling the location of competitive facilities by introducing a cover-based model and employed a branch and bound algorithm and several heuristic algorithms to solve it. Later, the same researchers presented three strategies for increasing the market share in a supply chain and compare the performance of these strategies with each other.<sup>44</sup> As reviewed, a large part of previous researches has focused on locating facilities in a monopoly environment. While considering such an assumption into reality is less feasible and, when locating and allocating facilities or new health centers, considering the competitive environment is more realistic. Therefore, in this research, the aim is to provide a model for locating new facilities in a competitive environment. For this purpose, the Nash bargaining strategy was used to formulate the problem, and the solutions obtained from solving the competitive model were compared with solutions of the monopoly model.

The present study is organized as follows: In section 2, a short review of Nash bargaining game is described to illustrate the usefulness of applying it when modeling competitive facility location problems. In section 3, the general assumptions of the proposed model are presented. Then, the model formulation is presented in section 4. An applied approach for solving the multi-objective model is described. A practical case study is presented in section 5. Finally, sensitivity analysis is done on important parameters in section 6 and conclusions are described in section 7.

## Methods

### Nash Bargaining Game

The goal of the Nash bargaining game, as a cooperative game, is dividing the benefits between two players based on their competition. This model requires that the feasible set be compact and convex.<sup>45,46</sup> However, some papers deal with the extensions of Nash bargaining theory to non-convex problems such as Denicolò and Mariotti,<sup>47</sup> and Zhou<sup>48</sup> and Rezaee.<sup>49</sup> Furthermore, the feasible set contains payoff vectors, with each individual payoff being greater than the breakdown payoff. The bargaining game can be defined with the triple  $(N, S, \bar{b})$ , which respectively specify the number of participating individuals, feasible set and breakdown points. The breakdown point is the payoff obtained whenever a player decides not to bargain with the other one. It  $u_1$  is defined as the utility function for the first player and  $u_2$  is defined as the utility function for the second player, the purpose of the Nash bargaining game is to maximize the following problem:

$$\underset{u \in S, u \geq b}{\overset{\rightarrow}{\text{Max}}} \prod_{i=1}^2 (u_i - b_i)$$

Where  $b_1$  and  $b_2$ , are the utilities obtained if one player decides not to bargain with the other one Nash.<sup>50,51</sup>

### General Assumptions

Non-competitive facility location models have a voluminous literature, considering the fact that it is more realistic to assume it must compete with other firms. Therefore, this study aims at presenting a new Competitive Location Model (CLM) based on Nash bargaining theory. First, a basic Non-Competitive Location Model (NCLM) is presented. Afterward, the CLM is extended by utilizing the prior NCLM and the Nash bargaining theory. In this regard, the Nash bargaining theory is applied to simulate the competitive environment. The presented model incorporates two objectives functions. The main purpose of this model is to maximize the total number of covered population by newly established facilities. The results obtained from solving both models (non-competitive and competitive) are compared to illustrate the advantages of the CLM in the following sections of this paper. We consider the establishment of a maximum number of  $N$  facilities to cover as much as possible the predetermined demand nodes, according to each facilities sphere of influence,  $D$  and other essential constraints. Therefore, some extra assumptions are considered before the model presentation:

1. Each facility healthcare center has a predetermined sphere of influence defined by  $D$ . It is assumed that all newly established facilities have the same sphere of influence.
2. A subset of potential candidate sites is considered according to the density of population centers.
3. Newly established healthcare centers could serve only a maximum amount of population defined by  $C$ , due to some natural capacity limitations for healthcare centers.
4. Newly established healthcare centers should cover at least a percent of the total population according to the NHS policies.
5. Euclidean distance measures are utilized between all locations. Note that without loss generality, we may use other measures instead of Euclidean distance to apply in the proposed model.
6. Only demand nodes, which are in the sphere of influence of newly established healthcare centers, are covered.

### Problem Formulation

In this section, the NCLM is introduced according to the necessary parameters and decision variable of the model.

### Non-Competitive Location Model

Parameters:

$i, I$	Set of potential candidate locations for establishing new healthcare centers
$j, J$	Set of demand nodes (population centers)
$d_{ij}$	Distance between demand node $j$ and potential candidate location $i$
$S_j$	The population of demand node $j$
$D$	Sphere of influence of the newly established healthcare centers
$N$	Maximum number of new healthcare centers which are permitted to be established
$a$	Minimum percentage of the total population which should be covered by all healthcare centers
$C$	The maximum amount of population which a newly established healthcare center is permitted to cover

Decision variables:

$y_i$	1, if a new healthcare center is decided to be established in candidate location $i$ ; 0 otherwise
$x_{ij}$	1, if demand node $j$ is covered by the newly established healthcare centers at $i$ ; 0 otherwise
$\delta_{il}$	Surplus variable assigns 1 whenever simultaneously at locations $i$ and $l$ new healthcare centers are decided to be established; 0 otherwise.

The objective (1) is to maximize the total covered population. The objective (2) aims at minimizing the sum of distances between newly established healthcare centers and the covered demand nodes. The constraints of the proposed model are as the following

$$\max z_1 = \sum_{i \in I} \sum_{j \in J} S_j x_{ij} \quad (1)$$

$$\min z_2 = \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \quad (2)$$

Constraint (3) shows that there is the maximum threshold for establishing new healthcare centers, which is defined by  $N$ . In other words, the total number of newly established healthcare centers should not exceed  $N$ , due to some budget limitations.

$$\sum_{i \in I} y_i \leq N \quad (3)$$

Constraint (4) determines that each population center could be covered by a maximum of one newly established healthcare centers.

$$\sum_{i \in I} x_{ij} \leq 1 \quad \forall j \in J \quad (4)$$

Constraint (5) restricts the assignment of population centers to newly established healthcare centers with sufficient sphere of influence.

$$Dy_i \geq d_{ij}x_{ij} \quad \forall i \in I \quad \forall j \in J \quad (5)$$

Constraint (6) ensures that at least  $\alpha$  percent of the total population must be covered by the newly established healthcare centers. This constraint is considered according to managerial experiences and also preventing the second objective function from assigning zero values to all variables.

$$\sum_{i \in I} \sum_{j \in J} s_j x_{ij} \geq \alpha \sum_{j \in J} s_j \quad (6)$$

Constraint (7) shows that population centers are only assigned to newly established healthcare centers. In other words, whenever a healthcare center is decided to be established at location  $i$  ( $y_i = 1$ ), at least one demand node should be assigned to this healthcare center. Otherwise, the healthcare center would not be established at location  $i$  and  $y_i = 0$ .

$$y_i \leq \sum_{j \in J} x_{ij} \quad \forall i \in I \quad (7)$$

Constraint (8) serves as the capacity limitation of newly established healthcare centers. This constraint is essential to be considered according to the qualitative measures of the NHS. It means that a specific newly established healthcare center is restricted to cover population to the maximum of  $C$  persons.

$$\sum_{j \in J} s_j x_{ij} \leq C \quad \forall i \in I \quad (8)$$

Constraints (9) and (10) are conditional constraints. These constraints try to assign population centers to the nearest healthcare centers. For instance, if a demand node is in the sphere of influence of two newly established healthcare centers, the mentioned demand node is assigned to the nearest healthcare center. To clarify, according to constraint (9) demand node  $j$  is assigned to the newly established healthcare center at  $l$  due to proximity.

$$\text{If } y_l y_i = 1 \text{ and } d_{lj} \leq d_{ij} \text{ then } x_{ij} < x_{lj} \quad (9) \\ \forall i, l \in I, i \neq l$$

$$\text{If } y_l y_i = 1 \text{ and } d_{lj} > d_{ij} \text{ then } x_{ij} > x_{lj} \quad (10) \\ \forall i, l \in I, i \neq l$$

Finally, constraint (11) specifies variable types of the presented model.

$$y_i \in \{0,1\}, x_{ij} \in \{0,1\}, \delta_{il} \in \{0,1\}, \\ \forall i, l \in I, i \neq l, j \in J \quad (11)$$

It should be mentioned that constraints (9) and (10) are conditional constraints and should be converted to linear constraints before running the model. Therefore, constraints (9) and (10) are replaced with constraints (12) to (15).

$$y_i + y_l \geq 2\delta_{il} \quad \forall i, l \in I, i \neq l \quad (12)$$

$$y_i + y_l - \delta_{il} \leq 1 \quad \forall i, l \in I, i \neq l \quad (13)$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} \leq d_{ij} \quad (14)$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} > d_{ij} \quad (15)$$

Therefore, the NCLM after constraint modifications is presented as follows.

$$\max z_1 = \sum_{i \in I} \sum_{j \in J} s_j x_{ij}$$

$$\min z_2 = \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij}$$

Subject to:

$$\sum_{i \in I} y_i \leq N$$

$$\sum_{i \in I} x_{ij} \leq 1 \quad \forall j \in J$$

$$Dy_i \geq d_{ij}x_{ij} \quad \forall i \in I \quad \forall j \in J$$

$$\sum_{i \in I} \sum_{j \in J} s_j x_{ij} \geq \alpha \sum_{j \in J} s_j$$

$$y_i \leq \sum_{j \in J} x_{ij} \quad \forall i \in I$$

$$\sum_{j \in J} s_j x_{ij} \leq C \quad \forall i \in I$$

$$y_i + y_l \geq 2\delta_{il} \quad \forall i, l \in I, i \neq l$$

$$y_i + y_l - \delta_{il} \leq 1 \quad \forall i, l \in I, i \neq l$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} \leq d_{ij}$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} > d_{ij}$$

$$y_i \in \{0,1\}, x_{ij} \in \{0,1\}, \delta_{il} \in \{0,1\}, \\ \forall i, l \in I, i \neq l, j \in J$$

### Competitive Location Model

In this section, the CLM will be outlined by utilizing the Nash bargaining theory. In this regard, the CLM is

formulated according to the following parameters and decision variables. Whenever the factor of completion is considered in a location model, it is assumed that two different companies (e.g.  $A$  and  $B$ ) or players are in favor of establishing their own new healthcare centers. Therefore, the situation of monopoly is not considered and thus a company is not the only player in the market; other competitors or players exist which offer the same services. In this case, each player has its own potential candidate locations and is seeking to establish its healthcare centers in locations which maximize their utility. In such a situation the main objective is to maximize the two-person Nash product, which in turn maximizes the total number of patients covered by the newly established healthcare centers.

*Parameters:*

$i, I$	Set of potential candidate locations for establishing new healthcare centers by player $A$
$k, K$	Set of potential candidate locations for establishing new healthcare centers by player $B$
$j, J$	Set of demand nodes (population centers)
$d_{ij}$	Distance between demand node $j$ and potential candidate location $i$
$d_{kj}$	Distance between demand node $j$ and potential candidate location $k$
$S_j$	The population of demand node $j$
$D$	Sphere of influence of the newly established healthcare centers
$N$	Maximum number of new healthcare centers which are permitted to be established
$b$	Minimum percentage of the total population which should be covered by the newly established healthcare centers
$C_a$	The maximum amount of population which a type $A$ newly established healthcare center is permitted to cover
$C_b$	The maximum amount of population which a type $B$ newly established healthcare center is permitted to cover
$\theta_a$	The breakdown point (disagreement payoff) for player $A$
$\theta_b$	The breakdown point (disagreement payoff) for player $B$

*Decision variables:*

$y_i$	1, if a type $A$ new healthcare center is decided to be established in candidate location $i$ ; 0 otherwise
$y_k$	1, if a type $B$ new healthcare center is decided to be established in candidate location $k$ ; 0 otherwise
$x_{ij}$	1, if demand node $j$ is covered by the type $A$ newly established healthcare centers at $i$ ; 0 otherwise
$x_{kj}$	1, if demand node $j$ is covered by the type $B$ newly established healthcare centers at $k$ ; 0 otherwise
$\delta_{il}$	Surplus variable and is assigned 1 whenever simultaneously at locations $i$ and $l$ type $A$ new healthcare centers are decided to be established; 0 otherwise
$\delta_{km}$	Surplus variable and is assigned 1 whenever simultaneously at locations $k$ and $m$ type $B$ new healthcare centers are decided to be established; 0 otherwise

The first objective is to maximize the game value between players  $A$  and  $B$ . Indeed, the goal of Nash bargaining game as a cooperative game is dividing of benefits between players  $A$  and  $B$  based on their competition. In this regard, the first objective incorporates two components of  $(\sum_i \sum_j s_j x_{ij} - \theta_a)$  and  $(\sum_k \sum_j s_j x_{kj} - \theta_b)$  which are multiplied by each other. To better explain, consider players  $A$  and  $B$ , whose preferences over outcomes are given by the utility functions  $(\sum_i \sum_j s_j x_{ij} - \theta_a)$  and  $(\sum_k \sum_j s_j x_{kj} - \theta_b)$ , respectively. In other words, these mentioned utility functions determine the total covered population by players  $A$  and  $B$ , respectively. Moreover,  $\theta_a$  is the breakdown point (threshold disagreement payoff) for player  $A$  and is defined as the starting point for bargaining for player  $A$ . To clarify, disagreement payoff for player  $A$  represents the possible payoff obtained if he/she decides not to bargain with player  $B$  or vice versa. This definition is similar for  $\theta_b$  (threshold disagreement payoff for player  $B$ ). It is worth mentioning  $\theta_a$  and  $\theta_b$  values are determined by management. Therefore, the first objective aims at maximizing the two-person Nash product.

$$\max z_1^c = (\sum_i \sum_j s_j x_{ij} - \theta_a) \times (\sum_k \sum_j s_j x_{kj} - \theta_b) \quad (16)$$

The second objective minimizes the sum of distances between population centers and the newly established healthcare centers (facilities). This objective includes two components: (i) the first minimizes the sum of distances

between population centers (demand nodes) and type *A* newly established healthcare centers (facilities). (ii) While the second minimizes the sum of distances between population centers (demand nodes) and type *B* newly established healthcare centers (facilities).

$$\min z_2^c = \sum_i \sum_j d_{ij} x_{ij} + \sum_k \sum_j d_{ik} x_{kj} \quad (17)$$

Constraints (18) and (19) specify that the utility functions of each player (*A* or *B*) should be greater than their specific breakdown points. As it was explained, breakdown points determine the possible payoff pairs obtained if one player decides not to bargain with the other player. In our location model breakdown points are defined as the least possible total population which each player is eager to cover. Therefore, the total population covered by each player will never be less than their breakdown points. In other words, the defined breakdown points are the least acceptable population for players *A* and *B* to start the game, respectively. Hence, player *A* and *B* will continue competing until the two-person (player) Nash product is maximized and the benefits are divided between them.

$$\sum_i \sum_j s_i x_{ij} \geq \theta_a \quad (18)$$

$$\sum_k \sum_j s_k x_{kj} \geq \theta_b \quad (19)$$

Constraint (20) is the maximum number of new healthcare centers which is defined by *N*. In other words, the total number of newly established healthcare centers by players *A* and *B* should not exceed *N*, due to some budget limitations.

$$\sum_i y_i + \sum_k y_k \leq N \quad (20)$$

Constraint (21) determines that each population center could be covered by a maximum of one newly established healthcare center. It means if a specific demand node is supposed to be covered it must be covered by only one of the players, (*A* or *B*).

$$\sum_i x_{ij} + \sum_k x_{kj} \leq 1 \quad \forall j \in J \quad (21)$$

Constraint (22) ensures that at least  $\alpha$  percent of the total population must be covered by the newly established healthcare centers. This constraint is considered according to managerial experiences and preventing the second objective function from assigning zero values to all variables.

$$\sum_i \sum_j x_{ij} s_i + \sum_k \sum_j x_{kj} s_k \geq \alpha \sum_j s_j \quad (22)$$

Constraints (23) and (24) show that population centers

are only assigned to newly established health centers. In other words, whenever a healthcare center is to be established at location *i* ( $y_i = 1$ ), at least one demand node should be assigned to this healthcare center. Otherwise, the healthcare center would not be established at location *i* and  $y_i = 0$ . This definition is the same for newly established healthcare centers by player *B*, specified by constraint (24).

$$y_i \leq \sum_j x_{ij} \quad \forall i \in I \quad (23)$$

$$y_k \leq \sum_j x_{kj} \quad \forall k \in K \quad (24)$$

Constraints (25) and (26) serve as the capacity limitation of newly established healthcare centers. This constraint is essential to be considered according to the qualitative measures of the NHS. It means that a specific newly established healthcare center is restricted to cover population to the maximum of  $C_a$  by player *A*, and  $C_b$  by player *B*.

$$\sum_j s_i x_{ij} \leq C_a \quad \forall i \in I \quad (25)$$

$$\sum_j s_k x_{kj} \leq C_b \quad \forall k \in K \quad (26)$$

Constraint (27) and (28) restrict the assignment of population centers to newly established healthcare centers with sufficient sphere of influence.

$$d_{ij} y_i \geq d_{ij} x_{ij} \quad \forall i \in I, j \in J \quad (27)$$

$$d_{kj} y_k \geq d_{kj} x_{kj} \quad \forall k \in K, j \in J \quad (28)$$

Constraints (29) until (32) are conditional constraints. These constraints try to assign population centers to the nearest healthcare centers. For instance, if a demand node is in the sphere of influence of two newly established healthcare centers, the mentioned demand node is assigned to the nearest healthcare center. To clarify, according to constraint (29) demand node *j* is assigned to the newly established healthcare center at *l* due to proximity. Constraints (31) and (32) are the exact same conditional constraints for newly established healthcare centers by player *B*.

$$\text{If } y_l y_i = 1 \text{ and } d_{lj} \leq d_{ij} \text{ then } x_{ij} < x_{lj} \quad (29) \\ \forall i, l \in I, i \neq l$$

$$\text{If } y_l y_i = 1 \text{ and } d_{lj} > d_{ij} \text{ then } x_{ij} > x_{lj} \quad (30) \\ \forall i, l \in I, i \neq l$$

$$\text{If } y_m y_k = 1 \text{ and } d_{mj} \leq d_{kj} \text{ then } x_{kj} < x_{mj} \quad (31) \\ \forall k, m \in K, k \neq m$$

If  $y_i y_m = 1$  and  $d_{mj} > d_{kj}$  then  $x_{kj} > x_{mj}$   
 $\forall k, m \in K, k \neq m$

Finally, constraint (33) specifies variable types of the presented model.

$$y_i \in \{0,1\}, x_{ij} \in \{0,1\}, y_k \in \{0,1\}, x_{kj} \in \{0,1\}, \delta_{il} \in \{0,1\}, \delta_{km} \in \{0,1\} \quad (33)$$

$$\forall i, l \in I, i \neq l, \forall k, m \in K, k \neq m, \forall j \in J$$

It should be mentioned that constraints (29) to (32) are conditional constraints and should be converted to linear constraints before running the model. Therefore, constraints (29) to (32) are replaced with constraints (34) to (41).

Constraints (34) to (37) are used to linearize constraints (29) and (30) and allocate the shortest path to a population center to access the nearest healthcare center type A. Also, constraints (38) to (41) are used to linearize constraints (31) and (32) and allocate the shortest path to a population set to access the healthcare center type B.

$$y_i + y_l \geq 2\delta_{il} \quad \forall i, l \in I, i \neq l \quad (34)$$

$$y_i + y_l - \delta_{il} \leq 1 \quad \forall i, l \in I, i \neq l \quad (35)$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{ij} \leq d_{lj} \quad (36)$$

$$x_{lj} - x_{ij} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} > d_{ij} \quad (37)$$

$$y_k + y_m \geq 2\delta_{km} \quad \forall k, m \in K, k \neq m \quad (38)$$

$$y_k + y_m - \delta_{km} \leq 1 \quad \forall k, m \in K, k \neq m \quad (39)$$

$$x_{kj} - x_{mj} \leq \delta_{km} \quad \forall k, m \in K, k \neq m, \quad | \quad d_{mj} \leq d_{kj} \quad (40)$$

$$x_{mj} - x_{kj} \leq \delta_{km} \quad \forall k, m \in K, k \neq m, \quad | \quad d_{mj} > d_{kj} \quad (41)$$

Therefore, the CLM after constraint modifications is presented as follows.

$$\max z_1^c = \left( \sum_i \sum_j s_i x_{ij} - \theta_a \right) \times \left( \sum_k \sum_j s_k x_{kj} - \theta_b \right)$$

$$\min z_2^c = \sum_i \sum_j d_{ij} x_{ij} + \sum_k \sum_j d_{kj} x_{kj}$$

Subject to:

$$\sum_i \sum_j s_i x_{ij} \geq \theta_a$$

$$\sum_k \sum_j s_k x_{kj} \geq \theta_b$$

$$\sum_i y_i + \sum_k y_k \leq N$$

$$\sum_i x_{ij} + \sum_k x_{kj} \leq 1 \quad \forall j \in J$$

$$\sum_i \sum_j x_{ij} s_j + \sum_k \sum_j x_{kj} s_j \geq \alpha \sum_j s_j$$

$$y_i \leq \sum_j x_{ij} \quad \forall i \in I$$

$$y_k \leq \sum_j x_{kj} \quad \forall k \in K$$

$$\sum_j s_j x_{ij} \leq C_a \quad \forall i \in I$$

$$\sum_j s_j x_{kj} \leq C_b \quad \forall j \in J$$

$$Dy_i \geq d_{ij} x_{ij} \quad \forall i \in I, j \in J$$

$$Dy_k \geq d_{kj} x_{kj} \quad \forall k \in K, j \in J$$

$$y_i + y_l \geq 2\delta_{il} \quad \forall i, l \in I, i \neq l$$

$$y_i + y_l - \delta_{il} \leq 1 \quad \forall i, l \in I, i \neq l$$

$$x_{ij} - x_{lj} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{ij} \leq d_{lj}$$

$$x_{lj} - x_{ij} \leq \delta_{il} \quad \forall i, l \in I, i \neq l, \quad | \quad d_{lj} > d_{ij}$$

$$y_k + y_m \geq 2\delta_{km} \quad \forall k, m \in K, k \neq m$$

$$y_k + y_m - \delta_{km} \leq 1 \quad \forall k, m \in K, k \neq m$$

$$x_{kj} - x_{mj} \leq \delta_{km} \quad \forall k, m \in K, k \neq m, \quad | \quad d_{mj} \leq d_{kj}$$

$$x_{mj} - x_{kj} \leq \delta_{km} \quad \forall k, m \in K, k \neq m, \quad | \quad d_{mj} > d_{kj}$$

$$y_i \in \{0,1\}, x_{ij} \in \{0,1\}, y_k \in \{0,1\}, x_{kj} \in \{0,1\}, \delta_{il} \in \{0,1\}, \delta_{km} \in \{0,1\} \\ \forall i, l \in I, i \neq l, \forall k, m \in K, k \neq m, \forall j \in J$$

It is clear that the two objective functions  $Z_1$  and  $Z_2$  in the NCLM (as same as  $Z_1^c$  and  $Z_2^c$  in the CLM) act in a different way and conflict with each other. In this case, individually optimizing the objectives leads to different solution sets. In order to cope with such a problem, the Global Criterion Method (GCM) as a strategy where the optimal solution is found by minimizing a preselected global criterion,<sup>52</sup>  $Z_3$ , such as the sum of the weighted relative deviations of the individual objective functions from the feasible ideal solutions. The GCM formulation is given by:

$$\text{Min } Z_3 = w_1 \times \frac{Z_1^* - Z_1}{Z_1^*} + w_2 \times \frac{Z_2 - Z_2^*}{Z_2^*} \quad (42)$$

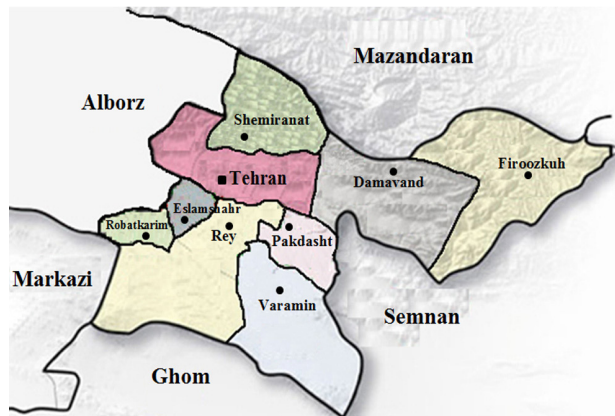
Where  $Z_3$  is the global criterion,  $Z_i^*$  are the target (ideal solutions) defined for the  $i$ th objective,  $Z_i$  is objective functions and  $W_i$  are the predefined weights of each objective. Thus, with targets defined for each objective, the multiple objectives are combined into an only function, which becomes the global optimization function for the process. It should be mentioned that the sum of the predefined weights is equal one and the exact value of these weights is defined by the decision-maker. To clarify, the greater the assigned weight of a specific objective (for instance  $W_2$ ) the closer the GCM solution to the solution obtained by individually optimizing the mentioned objective ( $Z_2$ ). Therefore, the model is reformulated after applying the global criterion method and is presented as follows for both NCLM ( $Z_3$ ) and CLM ( $Z_3^c$ ), by considering the previous constraints again.

$$\text{Min } z_3 = w_1 \times \frac{z_1^* - z_1}{z_1^*} + w_2 \times \frac{z_2 - z_2^*}{z_2^*} \Rightarrow \text{Min } z_3 = w_1 \times \frac{z_1^* - \left[ \sum_{i \in I} \sum_{j \in J} s_i x_{ij} \right]}{z_1^*} + w_2 \times \frac{\left[ \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \right] - z_2^*}{z_2^*} \quad (43)$$

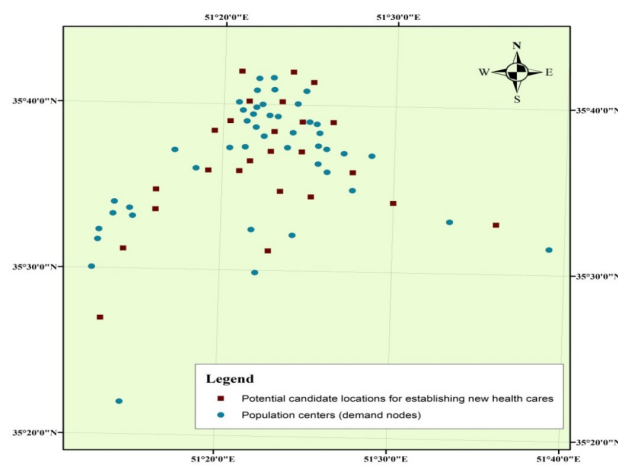
$$\begin{aligned}
 \text{Min } z_3^c &= w_1 \times \frac{z_1^c - z_1^a}{z_1^c} + w_2 \times \frac{z_2^c - z_2^a}{z_2^c} \Rightarrow \\
 \text{Min } z_3^c &= w_1 \times \frac{z_1^c - \left[ \left( \sum_{i,j} s_{ixj} - \theta_a \right) \times \left( \sum_{k,j} s_{kxj} - \theta_b \right) \right]}{z_1^c} + w_2 \times \frac{\left[ \sum_{i,j} d_{ixj} + \sum_{k,j} d_{kxj} \right] - z_2^c}{z_2^c} \quad (44)
 \end{aligned}$$

**Practical case study**

In this section, the model is applied to a set of population centers which need healthcare services and should be covered by some newly established healthcare centers. They are a population that is not covered by existing health centers. The mentioned case study is located in Tehran province of Iran and is illustrated in Figure 1. A geographical map, the population centers, and potential candidate locations for establishing new healthcare centers are depicted in Figure 2. On the other hand, for the governmental structure of healthcare in Iran, they cannot employ or dismissal employees easily. As a result, there is a limitation for visiting patients and each patient is assigned to a health center according to the distance between the health center and living location. If it is needed, patients are referred to specialized medical centers.



**Figure 1.** Geographical Map of Actual Application.



**Figure 2.** Population Centers and Potential Candidate Locations for Establishing New Healthcare Centers.

Furthermore, the complementary information on the mentioned case study is shown in Table 1. According to the population density of 45 demand nodes in the study area, 25 potential candidate locations for establishing new healthcare centers were defined. Besides, in the CLM, the 25 potential candidate locations were divided between player A and B in a balanced way. Therefore, 12 potential candidate locations were considered for player A and 13 potential candidate locations were considered for player B.

**Results and Discussion**

**Computational results of the NCLM**

In this section, the proposed the NCLM is solved and analysis is done on obtained results after determining the value of input parameters as follows. The sphere of influence of the newly established healthcare centers is considered to be 15 km, maximum number of new healthcare centers permitted to be established are 12, minimum percentage of the total population which should be certainly covered by the newly established healthcare centers is defined 70% and finally maximum amount of population which a newly established healthcare center is permitted to cover is considered to be 200000 persons. The proposed the NCLM was run using Lingo 15 and the illustrative results are gathered in Figure 3. To clarify, Figure 3 incorporates 4 maps entitled by a, b, c and d. Figure 3a illustrates the coordinates of 45 population centers and the numbered 25 proposed potential candidate locations. Figures 4b, 4c, and 4d depict the results obtained by solving the NCLM by considering the first objective function individually, the second objective function individually and finally by considering the global criterion function, respectively.

**Optimizing the First Objective in the NCLM**

In this case, the optimum value of the first objective is obtained 1669167. Actually, this means that 1669167 persons are covered by establishing 12 new healthcare centers. In other words, according to the constraint (3), 12 new healthcare centers (up to the maximum threshold) are selected among 25 potential locations to establish new healthcare centers. The comprehensive results of the NCLM are gathered in Table 2. According to this table, by individually optimizing the NCLM, 38 population centers are covered, overall. Seven population centers, namely 12, 22, 24, 26, 27, 43 and 44 were not covered by the newly established healthcare centers. These seven uncovered population centers have an overall population of 280087 persons. For a more precise and detailed perception of the obtained results, it is recommended to refer to Table 2. For instance, by considering Table 2, it can be seen that potential candidate location 2 has been selected



**Table 1.** Complementary information on the study area

Real-world application region: South of Tehran, Eslamshar and Rey			
least population among demand nodes	8043 (persons)	Longitude range of the region	51.21 ° -51.652 °
average population of demand nodes	43316 (persons)	Latitude range of the region	35.366 ° -35.694 °
greatest population among demand nodes	77792 (persons)	Number of existing population centers	45
Standard deviation of demand nodes	14009 (persons)	Total population of demand nodes	1949245 (persons)

**Table 2.** Selected candidate locations along with the covered demand nodes in detail: obtained from the NCLM

$Z_3$				$Z_2$				$Z_1$			
Selected candidate locations ( $y_i$ )	Number of covered demand nodes ( $x_{ij}$ )	Covered population	detailed covered demand nodes	Selected candidate locations ( $y_i$ )	Number of covered demand nodes ( $x_{ij}$ )	Covered population	detailed covered demand nodes	Selected candidate locations ( $y_i$ )	Number of covered demand nodes ( $x_{ij}$ )	Covered population	detailed covered demand nodes
1	3	133327	36-37-38	1	3	133327	36-37-38	1	4	176937	35-36-37-38
2	3	146785	3-5-9	7	3	143371	10-13-15	2	4	154828	1-3-5-9
7	3	143371	10-13-15	10	1	59136	34	7	3	143371	10-13-15
10	2	118065	17-34	12	6	199075	23-29-30-31-32-45	10	1	46492	14
12	6	119075	23-29-30-31-32-45	15	2	88806	6-7	12	7	195624	29-30-31-32-33-42-45
15	3	137672	4-6-7	16	3	122813	25-28-39	15	3	137672	4-6-7
16	4	148077	25-28-33-39	17	1	61125	8	16	4	181949	25-2834-39
17	2	107953	2-8	18	1	57852	44	17	2	107953	2-8
18	1	57852	44	20	3	180331	17-20-35	20	3	172958	18-20-40
20	3	149212	20-35-40	21	4	192678	14-16-18-21	21	3	137759	16-17-21
21	4	192678	14-16-18-21	22	1	44990	22	23	2	117594	23-41
23	2	88879	41-42	23	2	88879	41-42	24	2	96030	11-19
sum	36	1622946	-	sum	30	1372383	-	sum	38	1669167	-

( $y_2 = 1$ ) and it covers 4 population centers of 1, 3, 5 and 9 with an overall population of 154828 persons. To receive an illustrative perception of Table 2 it is recommended to investigate the results of Table 2 (contents under  $Z_1$ ) besides the related Figure 3b. According to Figure 3b, the selected candidate locations are illustrated with cross-hatched red squares, the covered population centers with blue circles, the uncovered population centers with yellow circles and finally, the unselected candidate locations are illustrated with red squares.

### Optimizing the Second Objective in the NCLM

In this case, the optimum value of the second objective is 50.7619 km. Indeed, the second objective aims at minimizing the sum of distances between the covered population centers and the newly established healthcare

centers. In this regard, according to the obtained solution set, 1372383 persons are covered by the newly established healthcare centers. To clarify, the noticeable decrease in the total covered population (in comparison with the solution set obtained by individually optimizing  $Z_1$ ), is due to the different performance of objectives  $Z_1$  and  $Z_2$  in assigning values to model variables ( $y_i$  and  $x_{ij}$ ). Hence, 1372383 persons are covered by establishing 12 new healthcare centers. In other words, according to constraint (3) 12 new healthcare centers (up to the maximum threshold) is selected among 25 potential locations to establish new healthcare centers. The comprehensive results of the NCLM are gathered in Table 2. According to this table (contents under  $Z_2$ ), by individually optimizing the NCLM, 30 population centers are covered, overall.

Fifteen population centers, namely 1, 2, 3, 4, 5, 9, 11,

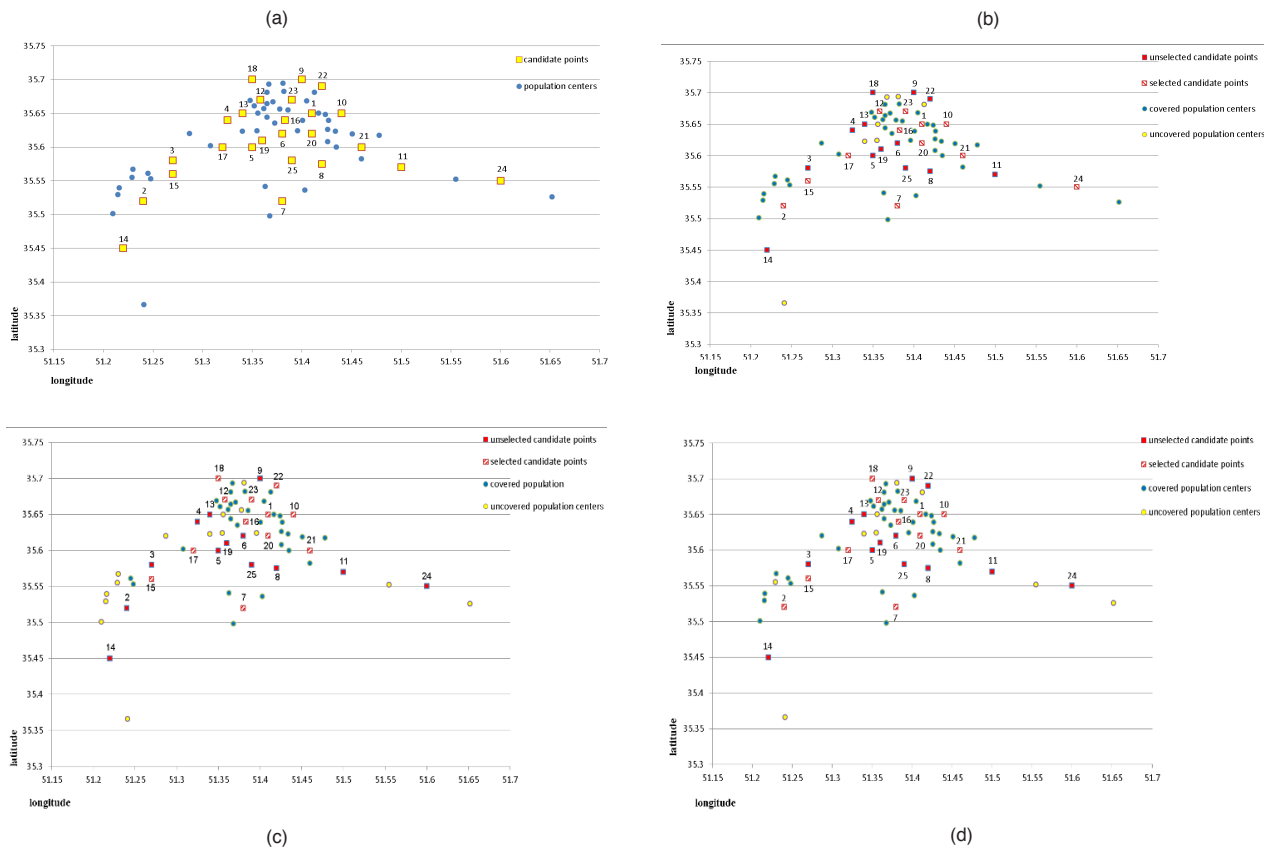


Figure 3. Geographical coordinates and illustrative results obtained by solving the NCLM

12, 19, 24, 26, 27, 33, 40 and 43 were not covered by the newly established healthcare centers. These fifteen uncovered population centers have an overall population of 576862 persons. For a more precise and detailed perception of the obtained results, it is recommended to refer to Table 2 (contents under  $Z_2$ ). For instance, by considering Table 2 it could be understood that potential candidate location 7 has been selected ( $y_7=1$ ) and it has covered 3 population centers of 10, 13 and 15 with an overall population of 143371 persons. To receive an illustrative perception of Table 2 it is recommended to investigate the results of Table 2 (contents under  $Z_2$ ) besides the related Figure 3c. According to Figure 3c, the selected candidate locations are illustrated with cross-hatched red squares, the covered population centers with blue circles, the uncovered population centers with yellow circles and finally, the unselected candidate locations are illustrated with red squares.

### Optimizing the Global Criterion Function in the NCLM

This time the global criterion function ( $Z_3$ ) is optimized after defining the weights of  $Z_1$  and  $Z_2$  ( $W_1$  and  $W_2$ ). Indeed, greater weight is assigned to  $Z_1$  due to the importance of the first objective in comparison with the second objective ( $W_1=0.9$  and  $W_2=0.1$ ). In this case, the optimum value

of  $Z_3$  is obtained 246.15. In this regard, according to the solution set obtained by minimizing  $Z_3$ , the value of the first objective is 1622946 (persons) and the value of the second objective is 70.12 (km). As it was expected, the quality of  $Z_1$  and  $Z_2$  have been decreased in comparison with their optimum values obtained by individually optimizing them ( $Z_1^*=1669167$  and  $Z_2^*=50.76$ ). Hence, 1622946 persons are covered by establishing 12 new healthcare centers. In other words, according to constraint (3) 12 new healthcare centers (up to the maximum threshold) is selected among 25 potential locations to establish new healthcare centers. The comprehensive results of the NCLM are gathered in Table 2. According to this table (contents under  $Z_3$ ), by optimizing the global criterion function ( $Z_3$ ), 36 population centers are covered, overall.

Nine population centers, namely 1, 11, 12, 19, 22, 24, 26, 27 and 43 were not covered by the newly established healthcare centers. These nine uncovered population centers have an overall population of 326299 persons. For a more precise and detailed perception of the obtained results, it is recommended to refer to Table 2 (contents under  $Z_3$ ). For instance, by considering Table 2 it could be understood that potential candidate location 2 has been selected ( $y_2=1$ ) and it has covered 3 population centers of 3, 5 and 9 with an overall population of

146785 persons. To receive an illustrative perception of Table 2 it is recommended to investigate the results of Table 2 (contents under  $Z_3$ ) besides the related Figure 3d. According to Figure 3d, the selected candidate locations are illustrated with cross-hatched red squares, the covered population centers with blue circles, the uncovered population centers with yellow circles and finally, the unselected candidate locations are illustrated with red squares.

**Computational Results of the CLM**

In this section, the proposed the CLM is solved and the results analyzed after determining the value of input parameters as follows. The sphere of influence of the newly established healthcare centers is considered

to be 15 km (for both players the sphere of influence is the same), maximum number of new healthcare centers permitted to be established are 12, minimum percentage of the total population which should be certainly covered by the newly established healthcare centers is defined 70 percent, maximum amount of population which a newly established healthcare center is permitted to cover is considered to be 200 000 persons, the breakdown point for player A and B is defined 680 000 (persons) and 685 000 (persons), respectively. The proposed the CLM was run by lingo 15 and the illustrative results are gathered in Figure 4. To clarify, Figure 4 incorporates 4 maps entitled by a, b, c and d. Figure 4 illustrates the coordinates of 45 population centers and the number of 25 proposed potential candidate locations for player A and B (red

**Table 3.** Selected candidate locations along with the covered demand nodes in detail: obtained from the CLM

Player (company) type (i,k)	$Z_1^c$				Player (company) type (i,k)	$Z_2^c$				Player (company) type (i,k)	$Z_3^c$			
	Selected candidate locations (Y,Y')	Number of covered demand nodes (X,X')	Covered population	Detailed covered demand nodes		Selected candidate locations (Y,Y')	Number of covered demand nodes (X,X')	Covered population	Detailed covered demand nodes		Selected candidate locations (Y,Y')	Number of covered demand nodes (X,X')	Covered population	Detailed covered demand nodes
A	1	5	194791	22-25-36-38-40	A	1	3	133327	36-37-38	A	1	4	192300	36-38-39-41
	4	4	172716	2-23-24-25		7	2	104068	10-15		4	3	113887	2/24/2027
	6	5	185268	26-28-33-37-39		10	1	59136	34		5	2	86980	8-26
	9	4	185523	31-41-42-44		12	5	199316	23-27-31-32-45		6	5	176261	25-28-33-37-40
	11	2	96030	11-19		Sum	11	465847	-		9	4	179205	22-42-43-44
	12	5	148185	27-29-30-32-43		15	1	48683	6		12	6	199075	23-29-30-31-32-45
	Sum	25	982613	-		16	4	148077	25-28-33-39		Sum	24	947708	-
B	14	4	184276	3-5-9-12	B	17	1	61125	8	B	14	4	184276	3-5-9-12
	15	2	52795	1-10		18	1	57852	44		15	5	190467	1-4-6-7-10
	17	4	198797	4-6-7-8		20	3	180331	17-20-35		20	3	161675	17-34-35
	20	3	170102	18-34-35		21	4	192678	14-16-18-21		21	4	192678	14-16-18-21
	21	3	175547	17-20-21		22	1	44990	22		24	2	96030	11-19
	25	4	185115	13-14-15-16		23	3	135710	41-42-43		25	3	176411	13-15-20
	Sum	20	966632	-		Sum	18	869446	-		Sum	21	1001537	-
	Sum of A & B		1949245			Sum of A & B		1365293			Sum of A & B		1949245	

squares belong to player *A* and yellow squares belong to player *B*). On the other hand, Figures 4b, 4c and 4d depict the results obtained by solving the CLM, by considering the first objective function individually, the second objective function individually and finally by considering the global criterion function, respectively. Type *A* potential candidate locations are numbered from 1 to 12 and type *B* potential candidate locations are numbered from 13 to 25.

#### Optimizing the First Objective in the CLM

In this regard, the optimum value of the first objective is obtained  $0.8522550E+11$  which represents the two-person Nash product. According to the solution set obtained by individually optimizing the first objective of a CLM, 12 candidate locations are selected among 25 candidate locations to establish new healthcare centers. Furthermore, 6 of these selected candidate locations are type *A* and 6 of them are type *B*. In fact, according to the constraint (20), up to the maximum possible threshold, new healthcare centers have been established. Also, the new healthcare centers established by player *A* have covered 25 population centers with an overall population of 982613 persons. On the other hand, the new healthcare centers established by player *B* have covered 20 population centers with an overall population of 966632 persons. Therefore, it is obvious that the newly established healthcare centers have covered all 45 existent population centers, whether by player *A* or *B*.

The selected candidate locations along with the covered demand nodes on detail are gathered in Table 3. For instance, according to Table 3 (contents under  $Z_1^c$ ), potential candidate location 1 has been selected by player *A* to establish a new healthcare center ( $y_1 = 1$ ). Also, this newly established healthcare center by player *A* has covered 5 population centers of 22, 25, 36, 38 and 40 with an overall population of 194791 persons. On the other hand, the lower part of Table 3 (entitled with *B*) represents the candidate locations selected by player *B* along with the covered population centers. For instance, potential candidate location 14 has been selected by player *B* to establish a new healthcare center and four population centers of 3, 5, 9 and 12 have been covered by it with an overall population of 184276 persons (as it was explained before, potential candidate locations 13 to 25 belong to player *B*). For a better and illustrative perception of the results obtained by individually optimizing the first objective of the CLM, it is recommended to refer to Figure 4b. In this figure, the selected candidate location by player *A* is presented with cross-hatched red squares, the covered population centers by type *A* newly established healthcare centers with red circles, the selected candidate

locations by player *B* are presented with cross-hatched blue squares, the covered population centers by type *B* newly established healthcare centers with blue circles.

#### Optimizing the second objective in the CLM

In this regard, the optimum value of the second objective is 50.28 (km) which represents the sum of distances between the newly established healthcare centers and the assigned population centers to them. According to the solution set obtained by individually optimizing the second objective of CLM, 12 candidate locations are selected among 25 candidate locations to establish new healthcare centers. Furthermore, 4 of these selected candidate locations are type *A* and 8 of them are type *B*. In fact, according to the constraint (20), up to the maximum possible threshold, new healthcare centers have been established. Also, the new healthcare centers established by player *A* have covered 11 population centers with an overall population of 495847 persons. The new healthcare centers established by player *B* have covered 18 population centers with an overall population of 869446 persons. Therefore, the newly established healthcare centers cover 29 population centers, with an overall population of 1365293 persons. The selected candidate locations along with the covered demand nodes on detail are gathered in Table 3. For instance, according to Table 3 (contents under  $Z_2^c$ ), potential candidate location 1 has been selected by player *A* to establish a new healthcare center ( $y_1 = 1$ ). Also, this newly established healthcare center by player *A* has covered 3 population centers of 36, 37 and 38 with an overall population of 133327 persons.

On the other hand, the lower part of Table 3 (entitled with *B*) represents the candidate locations selected by player *B* along with the covered population centers. For instance, potential candidate location 15 has been selected by player *B* to establish a new healthcare center and only one population center number 6 is covered by it with an overall population of 48683 persons (as it was explained before, potential candidate locations 13 to 25 belong to player *B*). For a better and illustrative perception of the results obtained by individually optimizing the second objective of the CLM, it is recommended to refer to Figure 4c. In this figure, the selected candidate locations by player *A* are presented with cross-hatched red squares, the covered population centers by type *A* newly established healthcare centers with red circles, the selected candidate locations by player *B* are presented with cross-hatched blue squares, the covered population centers by type *B* newly established healthcare centers with blue circles. Also, the uncovered population centers are presented with yellow circles.

### Optimizing the global criterion function in the CLM

This time the global criterion function ( $Z_3^c$ ) is optimized after defining the weights of  $Z_1^c$  and  $Z_2^c$  ( $W_1$  and  $W_2$ ). Indeed, greater weight is assigned to  $Z_1^c$  due to the importance of the first objective in comparison with the second objective ( $W_1=0.9$  and  $W_2=0.1$ ). In this case, the optimum value of  $Z_3^c$  is obtained 201.36. In this regard, according to the solution set obtained by minimizing  $Z_3^c$ , all population centers were covered. Hence, 1949245 persons are covered by establishing 12 new healthcare centers. In other words, according to constraint (20) 12 new healthcare centers (up to the maximum threshold) is selected among 25 potential locations to establish new healthcare centers. The comprehensive results of the CLM are shown in Table 3. According to this table (contents under  $Z_3^c$ ), by optimizing the global criterion function ( $Z_3^c$ ), 45 population centers are covered, overall. For a more precise and detailed perception of the obtained results, it is recommended to refer to Table 3 (contents under  $Z_3^c$ ). For instance, by considering Table 3 it could be understood that potential candidate location 1 has been selected ( $y_1 = 1$ ) and it has covered 4 population centers of 36, 38, 39 and 41 with an overall population of 192300 persons. For an illustrative perception of Table 3, it is recommended to investigate the contents of Table 3 (contents under  $Z_3^c$ ) besides the related Figure 4d. According to Figure 4d, the selected candidate locations by player A are presented with cross-hatched red squares, the covered population centers by type A newly established healthcare centers with red circles, the selected candidate locations by player B are presented with cross-hatched blue squares, the covered population centers by type B newly established healthcare centers with blue circles.

A comparison of the results obtained from both the NCLM and the CLM is illustrated in Figure 5. According to Figure 5a, the amount of covered population by optimizing different objectives is depicted in a column chart for the sake of comparison. On the other hand, the sum of distances between the newly established healthcare centers and the assigned population centers is illustrated in another column chart in Figure 5b. By a precise study of the presented column charts, the impact of individually optimizing the objectives could be understood on the obtained solution sets. Furthermore, it could be understood that optimizing the global criterion function ( $Z_3$  or  $Z_3^c$ ) has made a balance between the solutions obtained by individually optimizing the objectives ( $Z_1$  and  $Z_2$  or  $Z_1^c$  and  $Z_2^c$ ). Since the main goal of the management is to maximize the coverage of the population. It is clear that the total covered population was considerably increased

in the CLM.

### Sensitivity Analysis

In this section, due to the importance of competition among players to establish their own healthcare centers (facilities), a sensitivity analysis is done on important parameters of the CLM when the global criterion function is optimized. Therefore, different values are assigned to the sphere of influence of newly established healthcare centers ( $D$ ) as a crucial parameter (Figure 6). Then, the impact of this change is examined on the amount of covered population. Also, the impact of changing the sphere of influence of newly established healthcare centers is examined on the sum of distances. Furthermore, the sensitivity of the weights ( $W_1$  and  $W_2$ ) assigned to single objectives ( $Z_1^c$  and  $Z_2^c$ ) in the global criterion function is analyzed by assigning different weights (Figure 7)

According to Figure 6a, a general increase in the covered population could be seen by enhancing the sphere of influence of newly established healthcare centers (except for moving from radius 10 to 11 and also from radius 15 to 16). The general increasing trend is natural and is due to the higher ability or freedom of newly established healthcare centers in covering more populated demand nodes. On the other hand, by precisely comparing Figure 6b with Figure 6a, it could be concluded that both charts have the same trend. Furthermore, the decrease in the amount of covered population for radius 11 and 16 could be explained by comparing the trend of both charts in the mentioned points. To better explain, when the global criterion function is optimized, a solution set is obtained which is far from the optimum solution sets obtained by individually optimizing the single objectives. In this case, by increasing sphere of influence from 11 to 12 (as same as from 15 to 16), unexpectedly the amount of covered population is decreased. This could be inferred as the stronger impact of the second objective in assigning closer demand nodes to newly established healthcare centers even less populated demand centers.

Figure 7 illustrates the chart obtained from considering different combinations of weights in the global criterion function and calculating the amount of covered population when the global criterion function is optimized. According to Figure 7, a general decreasing trend could be seen (except for weight combination 0.7-0.3 to 0.6-0.4). This global decrease in the covered population is due to the performance of the second objective in assigning lower populated demand nodes to newly established healthcare centers for the sake of proximity. To better explain, this chart illustrates the change in the amount of covered population when the weight of the second objective ( $W_2$ )

is continuously increasing in the global criterion function. Therefore, the general decreasing trend in the chart could be inferred as the impact and performance of the second objective. Furthermore, the amount of covered population after the weight combination of 0.5-0.5 is steady until the last weight combination, 0.1-0.9, with a population around 1 400 000. This steady amount of covered population could easily be concluded as constraint (22) performance in enforcing the model to cover at least 70 percent of the total population (about 1 364 471 persons). In other words, although the impact of the second objective increases by increasing the value of  $W_2$ , this is possible until constraint (22) is not violated. According to Figure 7, the combination of the changing weights in the competitive model from 0.5-0.5 to 0.1-0.9, does not have an effect on the constraint (22). Therefore, according to the results obtained from sensitivity analysis and managerial tradeoffs, the most desirable sphere of influence of the newly established healthcare centers was considered as 15 km ( $D=15$ ) in this paper.

## Conclusions

The location of facilities in healthcare is a considerable issue and location-allocation models have an undeniable role in making health centers more accessible to patients. In this paper, a CLM is proposed to cover the demand centers as much as possible in Tehran province, Iran. In this regard, the situation of competition among two players (companies) who are in favor of establishing their own health centers is simulated by applying Nash bargaining theory. The results obtained from applying the CLM revealed the abilities of this model in simulating the competitive situations. For instance, by comparing the results obtained from both models (non-competitive and competitive) it was clear that the total covered population was considerably increased in the CLM. The results of solving this proposed model can be used as a basis for decision making by managers. Because of any wrong decision, in addition to raising the costs of the health system, can also lead to irreparable damage to human and social health. The implementation of the case study has also confirmed the ability of the model in locating and allocating health services in Tehran's healthcare centers. In fact, due to the expansion of the private sector to the provision of health care, this research has tried to provide a mathematical approach to a competitive environment rather than providing a list of appropriate locations for the establishment of health centers to give the whole community the positive effects of this action. In fact, this study, considering two main objective functions including maximizing the total population covered and minimizing

the total distance of facilities from the newly established facility, can have an important impact on the distribution of health services as well as other urban services. In addition, this research can be used by managers to locate the healthcare centers of other cities as well.

The proposed model in this paper was a multi-objective optimization problem and is applicable in many cases. The presented model could be easily modified by adding extra objectives to suit its application area. Moreover, the model could be extended by allowing budget constraints such as considering an available budget for establishing new health centers. Another direction for future research could be defining the population areas with a uniform distribution around a specific point of the population area, instead of considering population areas to be aggregated demand nodes. Another extension route would be dividing the population of a demand center in the sphere of influence of two or more health centers, according to their proximity to the demand center. Developing a dynamic location model would also be an interesting research topic, which incorporates population changes and migration during the time.

## Abbreviations

(NHS): National Health Service; (NCLM): Non-Competitive Location Model; (CLM): Competitive Location Model; (GCM): Global Criterion Method.

## Competing interests

The authors declare no competing interests.

## Authors' contributions

H.M., M.J.R., M.S. jointly designed the study. H.M., M.J.R., S.Y. were involved in data collection and analysis. M.J.R., S.Y. made the major contribution in revising the manuscript. All authors read and approved the final manuscript.

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