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Research Article

An Inventory Model for Obsolescence Items with Consideration of Permissible Delay in Payments: Case Study of Obsolete Medicines in Pharmacies

Hassan Zamani Bajegani[1], Mohammad Reza Gholamian[1*]

[1]School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Abstract

Background and Objectives: In the real world, the obsolescent items are some items that lose their value over time due to the emergence of new technology. Because of rapid changes in technology, inventory management of such items is considered in recent years. Moreover, suppliers try to encourage the retailers for purchasing an item before it is outmoded with some policies such as discounts, rebates, bonus backs, and delayed payment and so on. Given the speed of medical advances in pharmaceutical industries and the successive release of new products to the market, and also the fact that delay in payment is the main marketing issue in selling the medicine products to the pharmacies in Iran, in this paper, the delay in payment policy for obsolescent items is studied and an inventory control model is developed to respond to these conditions.

Methods: The model minimizes total inventory cost to achieve the optimal cycle time with respect to the constant demand rate and sudden obsolescence with exponential distribution over time. Numerical examples referring to a real case study in the pharmaceutical industry like drugstores are given to demonstrate the performance of the model in different states of delay in payment.

Findings: Based on the results, according to the considered permissible payment period based on the actual market situation, the length of the optimal ordering cycle is usually smaller than that of the payment cycle. Besides, with the reduction of the expected lifetime, the inventory costs are concurrently reduced and on the other hand, the increase in the expected lifetime raises the inventory costs. Moreover, there is no exact relationship between credit time due date (i.e. payment time) and inventory cost; which shows the high sensitivity of this parameter in finding the optimal solutions.

Conclusions: An inventory control model for obsolescent items in the pharmaceutical retailing industry is introduced under delay in payment policy. In the presented model, after determining the inventory cost functions regarding obsolescence, holding and delay in payment costs, which are related to the pharmaceutical supply chain, the total cost function is introduced during the lifetime of items and the optimality is checked by convexity test through the second derivative in two cases of delay in payment in the model.

Keywords: Inventory Control; Delay in Payment; Obsolescence; Perishable Items;

Introduction

In recent years, many items in real markets, especially high-tech, are outdated after a few times with emerging new technologies. Moreover, some products encounter prohibition of the global environment about the use of particular commodities and become obsolescent. Additionally, there are some products, such as Valsartan, which become obsolescent after being identified as a harmful substance by the US Food and Drug Administration[1]. Under these critical conditions, it is very important to manage the inventory level of the items that may be obsoleted in the near future accurately. On the other hand, merchandising companies generally propose some incentive policies (particularly in financial aspects), which can encourage the buyers for more purchasing and consequently bring more profits for the company. In this paper, an application of these financial incentive policies is studied for the items that are subject to obsolescence.

*Corresponding Author: Mohammad Reza Gholamian
Email: Gholamian@iust.ac.ir

Considering that the studied industry in thisresearch is the pharmaceutical industry and especially the Iranian drugstores are focused, this research is concentrated on the most important financial issues of these retailers i.e. the delay in payment. Since, according to the laws of ministry of health, the price of products is fixed, and there is no possibility of reducing the sale price of medicines, the most important marketing issues that affect the sale of products by these retailers would be the quantity discounting and receiving longer permissible credit period of payment. As in the Iranian market, the products (especially medical products) are usually sold based on future trading by check, the topic of trade credit or in other words, delay in payment, is a real and critical issue in these markets and industries. Therefore, this topic is focused in this study. Unfortunately, while this financial aspect is widely used for deteriorating items but in obsolescence, there is no in-deep research as far as the authors are aware. In the following, the concepts and relevant researches are reviewed to demonstrate the research gap as much as possible.

Few studies have been conducted on pharmaceutical inventory control and supply chain management. Tsolakis and Srai used a system dynamics approach for inventory planning in the green pharmaceutical supply chain[2]. Saedi at al developed a stochastic model for inventory optimization in pharmaceutical supply chains considering the product shortage resulting from supply disruption[3]. Stecca at al. concerned the hospital's drug supply chain in which the products are purchased by a central hospital pharmacy and then distributed among the internal wards. The optimal inventory cost is determined in the central pharmacy[4].

Basically, in most of the articles with the topic of obsolescence, developments are considered in the model. Among such studies, Joglekar and Lee have developed the profit maximization model by considering sales price[5]. Cobbaert and Oudheusden proposed a model to control the products with the risk of fast and unexpected obsolescence. They have examined the model with fixed and variable obsolescence rates with and without shortage[6]. Arcelus et al offered a model with gradual obsolescence for maximizing the profit, where the demand is a function of sale's time and price[7]. Song and Lau offered a periodic inventory model for sudden obsolescence by considering the concept of dynamic programming[8]. In Wang and Tung, considering the discount in price, demand serves as a function of population growth during the product life cycle[9].

On the other hand, trade credit and delay in payment has been widely used in deteriorating inventory studies. Mahata developed an economic production quantity (EPQ) model with exponential deterioration in a three-level (i.e. supplier-retailer-customer) supply chain in which both supplier and retailer offer full and partial trade credits to their next level respectively[10]. Majumder et al proposed an EPQ deterioration model with crisp/fuzzy time-dependent demand and considered trade credit offered by both suppliers and retailers[11]. Shabani et al developed a new two-warehouse inventory model with fuzzy deterioration and fuzzy demand rates under conditionally permissible delay in payment[12]. Sharma presented an ordering model under partial trade credit and backlogging in a two-level supply chain, where the retailer can take full trade credit if s(he) pays a percentage of the purchasing costs[13]. Pourmohammad Zia and Taleizadeh developed a three-level model (i.e. supplier-retailer-customer) in which two types of partial payment (i.e. delayed and multiple advanced) will be granted to retailer if s/he pays the minimum purchasing cost[14]. Kumar et al developed an inventory model for the deteriorating products with permissible delay in payment under inflation, where the demand rate is considered as stock-dependent and the deterioration rate of each product follows Weibull distribution[15]. Tsao introduced an inventory control model for non-instantaneously deteriorating items under price adjustment and trade credit. The modeling of that paper is based on the price differentiation of the products by the retailer during the deteriorating period in comparison with the non-deteriorating period according to the status of the delayed time of payment and the starting time of the deterioration period[16]. Liao et al. examined an inventory control model considering the delay in payment, the limited capacity of the warehouse, and the opportunity of using the rented warehouse[17]. Modeling of the paper is also based on the retailer's credit period offered by the supplier and the customer's credit period offered by the retailer with respect to the personal and rental warehouses. Nematollahi et al proposed a collaboration model for the pharmaceutical supply chain with social responsibility under periodic review inventory control policy. In this article, a bi-objective model is developed that contains profit maximization along with maximizing the customer service level, to ensure that the lack of necessary medicines does not occur at all.[18]. Ebrahimi et al developed a model to coordinate a two-echelon supply chain with periodic review inventory control policy, based on “delay in payment” contract and considering retailers’ promotional effects[19]. Shah et al presented an EOQ model with a two-level “delay in payment” contract for perishable pharmaceutical products that have a maximum lifetime[20]. In this regard, Johari et al proposed a periodic review inventory model in a two-echelon supply chain considering “delay in payment” for the price credit-dependent demand and inflation levels[21].

Table 1: Categorization of the research topics in obsolete products

	Assumptions and decision-making policies											
	Decision variables			Delay in payment	Type of chain components		Number of chain levels		Demand		Type of product	
	Cycle time	Price	Order quantity		EPQ	EOQ	Multi echelons	One echelon	Certain	Uncertain	Obsolescence	Deterioration
Joglekar and Lee[5]			*			*		*	*		*	
Cobbaert and Oudheusden[6]			*			*		*	*		*	
Wang and Tung[7]	*	*				*		*		*	*	
Song and Lau[8]	*					*		*				
Persona et a[9]			*			*		*	*		*	
Mahata[10]	*			*	*		*		*			*
Majumder et al[11]	*			*		*		*		*		*
Shabani et al[12]	*			*		*		*	*			*

	Assumptions and decision-making policies											
	Decision variables			Delay in payment	Type of chain components		Number of chain levels		Demand		Type of product	
	Cycle time	Price	Order quantity		EPQ	EOQ	Multi echelons	One echelon	Certain	Uncertain	Obsolescence	Deterioration
Sharma[13]	*			*		*		*	*			*
Pourmohammad Zia and Taleizadeh[24]	*			*		*	*		*			*
Kumar et al[15]	*			*		*		*	*			*
Liao et al[16]	*			*		*		*	*			*
Tsao[17]	*			*		*		*		*		*
Chen and Teng[22]	*			*		*		*	*			*
This study	*			*		*		*	*		*	

In Table 1, the important studies are classified according to the key criteria discussed in the literature review. As shown in the table, unlike “delay in payment” researches on deteriorating items, there are not much conducted researches on using ‘delay in payment” and more generally, financial aspects on obsolete items. Therefore, this concept is considered as the main interest of this research.

The rest of this paper is organized as follows. In the next section, assumptions and notations are introduced and then the mathematical model is brought in section 3. Then, in section 4, the model is solved with real sample data of drugstore retailer. Meanwhile, to assess the impact of critical parameters on the model, the sensitivity analysis is performed based on a combination of parameter variations. Finally, the paper is concluded by commenting on the directions for future researches in section 5.

Methods

Assumption and notations

The following assumptions are considered in

this study:

- The annual demand rate is constant.
- A single item is considered.
- The obsolescence happens suddenly.
- Obsolescence lifetime is exponential.
- Planning horizon is infinite.

Meanwhile, the following notations have been used in developing the mathematical model.

Parameters:

t: The time that product becomes obsolete (Year)

L: Expected lifetime of the product (years)

R: demand per year.

A: The cost of ordering (Rs)

H: The holding cost percentage per unit

C_p: The cost of purchasing per unit (Rs)

C_s: Selling price of obsoleted products per unit (Rs)

M: The payment time after receipt the order (years)

I_e: The interest rate earned in a cycle

I_p: The interest rate charged in a cycle

P_s: The probability that obsolescence does not occur during the order cycle

The decision variables and objective functions

T: The optimal cycle time

T_1: The optimal cycle time whenever $T \geq M$

T_2: The optimal cycle time whenever $M > T$

C_{c1}: Total inventory cost including ordering and holding costs as well as interest earned payment and interest charged payment, minus obsolete salvage values, per cycle whenever $T \geq M$ (Rs)

C_{c2}: Total inventory cost including ordering and holding costs as well as interest earned payment, minus obsolete salvage values, whenever $M > T$ (Rs)

C_{L1}: Total inventory cost including ordering and holding costs as well as interest earned payment and interest charged payment, minus obsolete salvage values, per year whenever $T \geq M$ (Rs)

C_{L1}: Total inventory cost including ordering and holding costs as well as interest earned payment, minus obsolete salvage values, per year whenever $M > T$ (Rs)

C_L (M): Total inventory cost at the end of trade credit time (M) (Rs)

Problem statement

The basic framework of the problem is that the seller sells his product to the buyer but allows the buyer to pay with a delay after receiving the ordered products; the seller considers a credit period for the buyer. The earned and charged interest rates can greatly influence the allowable delay in payment, which is acceptable to both parties. Besides, the type of obsolescence is sudden obsolescence in which the items are outdated by the exponential distribution function during obsolescence lifetime, and after obsolescence, the remaining items can be sold by a salvage price.

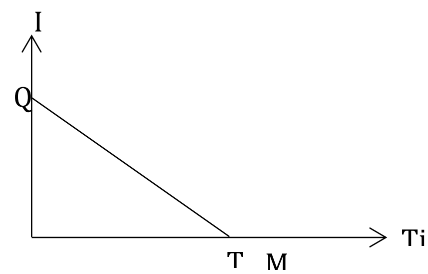
Mathematical Model

According to Joglekar and Lee[5] and Cobbaert and Oudheusden[6], the obsolescence salvage value at time t is the value of items, which become obsolescence at this time by taking an exponential probability distribution life time with mean L. So, considering T at each period, $(T-t)R$ items will be obsolete at time t ($0 < t < T$) and salvage value of obsoleted items can be determined as shown in equation (1).

$$CS = \int_0^T [(T-t)R](C_s)\left[\left(\frac{1}{L}\right)e^{-t/L}\right]dt = \left(RL\left[-1+e^{-\frac{T}{L}}\right]+TR\right)C_s \quad (1)$$

Since at time t ($0 < t < T$), TR - tR items are obsolete, the average inventory at this time would be $t.[T * R - \left(\frac{tR}{2}\right)]$; whilst for $t > T$ this value would be the same $T*R/2$.

Therefore, the holding costs associated with



these inventory values would be:

$$C_{H1} = \int_0^T \left[T * R - \left(\frac{tR}{2}\right)\right](t\, C_p\, H)\left[\left(\frac{1}{L}\right)e^{-t/L}\right]dt = \frac{HC_p[2LR(-L+T)+Re^{-\frac{T}{L}}(2L^2-T^2)]}{2} \quad (2)$$

$$C_{H2} = \int_T^\infty \left[\left(\frac{T*R}{2}\right)(T)\right](C_p\, H)\left[\left(\frac{1}{L}\right)e^{-t/L}\right]dt = \frac{H\, C_p\, R\, T^2\, e^{-\frac{T}{L}}}{2} \quad (3)$$

And generally:

$$C_H = C_{H1} + C_{H2} = H\, L\, C_p * \left(TR - RL\left[1-e^{-\frac{T}{L}}\right]\right) \quad (4)$$

By considering the delay in payment, new terms will be added to the model based on the priority relation of the length of credit period (M) and the length of product cycle (T) as discussed in the following:

Case 1: $T > M$

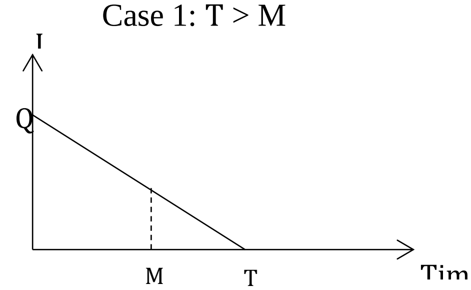


Figure 1: Inventory level in interval [0, T] at case M ≤ T

It is assumed that the obsolescence time (t) is occurred at [0, M]. Given that the probability of non-obsolescence up to time t is:

$$\int_t^{\infty} \frac{1}{L} e^{-t/L} = e^{-\frac{t}{L}} \tag{5}$$

The average interest earned is equal to:

$$IE_1 = C_p I_e \int_0^M t\, R \left(\frac{1}{L}\right) e^{-\frac{t}{L}} dt = C_p\, I_e\, R[L - e^{-\frac{M}{L}}(L+M)] \tag{6}$$

Similarly, the average interest paid can be calculated based on the inventory bought but not used at the interval [M, T] as follows:

$$IC_1 = C_p I_p \int_M^T I(t)dt = C_p I_p \int_M^T (T - t)R dt = C_p\, I_p \left[\frac{RT^2}{2} - (MTR - \frac{R\,M^2}{2})\right] \tag{7}$$

Case 2: M ≥ T

Figure 2: Inventory level in interval [0, T] at case M > T

In this case, the items are obsolete at time [T, ∞], so, no interest payable could be considered, but the interest earned can be explained with the following two terms:

$$IE_{21} = C_p I_e \int_0^T tR \left[\left(\frac{1}{L}\right) e^{-\frac{t}{L}}\right] dt = C_p I_e R\,[L - e^{-\frac{T}{L}}(L+T)] \tag{8}$$

$$IE_{22} = C_p\, I_e\, T R \int_T^M (t-T) \left[\left(\frac{1}{L}\right) e^{-\frac{t}{L}}\right] dt = C_p\, I_e\, TR \left[\left(-e^{-\frac{M}{L}}(L+M-T) + L\, e^{-\frac{T}{L}}\right)\right] \tag{9}$$

Considering the equations (1), (4), (6) and (7), the total cost will be summarized as equation (10) if T > M:

$$C_{c1} = A + C_H + IE_1 + IC_1 = A + (H\,L\,C_p - C_s)*(TR - R\,L\left[1 - e^{-\frac{T}{L}}\right]) + C_p\, I_p(\frac{RM^2}{2} - M\,TR + \frac{RT^2}{2}) - C_p\, I_e\, R\,[L - e^{-\frac{M}{L}}(L+M)] \tag{10}$$

Also, considering the equations (1), (4), (8) and (9), total cost will be summarized, as equation (11) if T ≤ M:

$$C_{c2} = A + C_H + IE_{21} + IE_{22} = A + (\mathrm{H\,L\,C_p} - \mathrm{C_s})\,[\mathrm{T} * \mathrm{R} - \mathrm{RL}\,(1 - \mathrm{e}^{-\frac{T}{L}})] - C_p\, I_e\, \mathrm{R} \left(\mathrm{L} - \mathrm{e}^{-\frac{T}{L}}(\mathrm{L}+\mathrm{T})\right) - C_p\, I_e\, \mathrm{TR} \left[\left(-e^{-\frac{M}{L}}(L+M-\mathrm{T}) + L\, e^{-\frac{T}{L}}\right)\right] \tag{11}$$

The equations can be extended into all cycles. Lemma 1 is useful in this regard.

Lemma 1: The average inventory cost over all cycles is obtained by:

$$\mathrm{C_{Li}} = \frac{C_{ci}}{1 - e^{-\frac{T}{L}}} \quad \mathrm{i} = 1, 2 \tag{12}$$

Proof: As the product lifetime function is exponential, and due to memory-less property of this function, the inventory cost at the beginning of each cycle would be the same as the previous cycle if the obsolescence does not happen. Therefore, the total inventory cost per the expected lifetime of the product would be:

$$\mathrm{C_{Li}} = \mathrm{C_{ci}} + \mathrm{C_{Li}}\,\mathrm{P_s} \Rightarrow \mathrm{C_{Li}} = \frac{\mathrm{C}_{ci}}{1 - \mathrm{P}_s} \quad \mathrm{i} = 1, 2 \tag{13}$$

On the other hand,

$$\mathrm{P_s} = \int_{\mathrm{T}}^{\infty} \left[\left(\frac{1}{\mathrm{L}}\right) e^{-\frac{\mathrm{t}}{\mathrm{L}}}\right] dt = e^{-\frac{T}{L}} \tag{14}$$

As a result:

$$\mathrm{C_{Li}} = \frac{\mathrm{C}_{ci}}{1 - e^{-\frac{T}{L}}} \quad \mathrm{i} = 1, 2 \tag{15}$$

Hereon, an average inventory cost during the expected lifetime of the product ($\mathrm{C_L}$) for both case T > M and M ≥ T is calculated based on equations (14) and (15) respectively:

$$\mathrm{C_{L1}} = \frac{\mathrm{A} + (\mathrm{H\,L\,C_p} - \mathrm{C_s}) * \left(\mathrm{TR} - \mathrm{R\,L}\left(1 - \mathrm{e}^{-\frac{T}{L}}\right)\right) - C_p\, I_e\, \mathrm{R}[\mathrm{L} - \mathrm{e}^{-\frac{M}{L}}(\mathrm{L}+\mathrm{M})] + C_p I_p \left(\frac{\mathrm{R\,M^2}}{2} - \mathrm{MTR} + \frac{\mathrm{R\,T^2}}{2}\right)}{1 - e^{-\frac{T}{L}}} \tag{16}$$

$$\mathrm{C_{L2}} = \frac{\mathrm{A} + (\mathrm{H\,L\,C_p} - \mathrm{C_s}) * \left(\mathrm{TR} - \mathrm{R\,L}\left(1 - \mathrm{e}^{-\frac{T}{L}}\right)\right) - C_p\, I_e\, \mathrm{R}\,[\mathrm{L} - \mathrm{e}^{-\frac{T}{L}}(\mathrm{L}+\mathrm{T})] - C_p\, I_e\, \mathrm{TR}\left[\left(-e^{-\frac{M}{L}}(L+M-\mathrm{T}) + L\, e^{-\frac{T}{L}}\right)\right]}{1 - e^{-\frac{T}{L}}} \tag{17}$$

In special cases that T = M, the interest payable cost would be:

$$\mathrm{IE} = C_p I_e \int_0^{\mathrm{T}} tR \left[e^{-\frac{\mathrm{t}}{\mathrm{L}}}\right] dt = C_p\, I_e\, \mathrm{R}\,[\mathrm{L} - \mathrm{e}^{-\frac{T}{L}}(\mathrm{L}+\mathrm{T})] \tag{18}$$

Consequently, the total cost per cycle and expected lifetime of the product is obtained as follows:

$$\mathrm{C_c} = \mathrm{A} + (\mathrm{H\,L\,C_p} - \mathrm{C_s}) * (\mathrm{TR} - \mathrm{R\,L}[1 - \mathrm{e}^{-\frac{T}{L}}]) - C_p\, I_e\, \mathrm{L\,R}(\mathrm{L} - \mathrm{e}^{-\frac{T}{L}}[\mathrm{L}+\mathrm{T}]) \tag{19}$$

$$\mathrm{C_L} = \frac{\mathrm{A} + (\mathrm{H\,L\,C_p} - \mathrm{C_s}) * \left(\mathrm{TR} - \mathrm{RL}\left(1 - \mathrm{e}^{-\frac{T}{L}}\right)\right) - C_p I_e\, \mathrm{L\,R}(\mathrm{L} - \mathrm{e}^{-\frac{T}{L}}[\mathrm{L}+\mathrm{T}])}{1 - \mathrm{e}^{-\frac{T}{L}}} \tag{20}$$

The convexity of the model, (i.e., $\frac{\partial^2 C_L}{\partial^2 T} > 0$), is given in appendix A (for the case M ≤ T) and appendix B (for the case M > T), respectively.

Meanwhile, the following algorithm is used to determine the optimal solutions. It is assumed that T_1 is the optimal ordering cycle in the first case (i.e. $M \leq T$) and T_2 is the optimal ordering cycle in the second case (i.e. $M > T$).

Step 1: If $T_1 > M$ & $T_2 < M$) then compare $C_L(T_1)$ and $C_L(T_2)$ and go to 4:

Step 2: If ($T_1 > M$ & $T_2 \nless M$) then compare $C_L(T_1)$ and $C_L(M)$ and go to 4;

Step 3: If ($T_1 \ngtr M$ & $T_2 < M$) then compare $C_L(T_2)$ and $C_L(M)$ and go to 4;

Step 4: Select T^{*}.

Step 5: If ($T_1 \ngtr M$ & $T_2 \nless M$) then set $T^* = M$.

The algorithm provides the optimal value of T in all cases and then, the optimal order quantity (Q*) is determined through T*.

Results

5.1. Numerical Examples

In this study, a drugstore that sells a wide range of typical medicines, which are exposed to sudden obsolescence (such as Valsartan) has been selected as the case study. The store has been operated as a retailer in the field of medicine sales since 2011.

Let R = 100000, A = 400000, C_p = 20000, C_s =5000, H= 0.2, I_p = 0.15, I_e = 0.13, M = 0.4, in this case study. First, the problem was to be solved mathematically. However, because of the complexity of conveying proof using the second derivative, the numerical results were run with distinct time values (T) in the real range [0, 1]. Based on the introduced method, the corresponding optimal values are obtained as illustrated in Figures 3 and 4. In this case, it is assumed that the cost of obsolescence per unit (C_s) is simply equal to ten percent of the cost of purchasing per unit (C_p). The computational results show that the cost functions are convex under the examined T values.

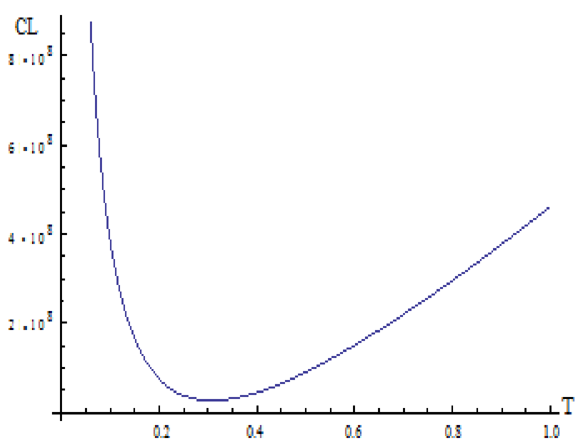


Figure 3: Inventory cost function at the case $M \leq T$

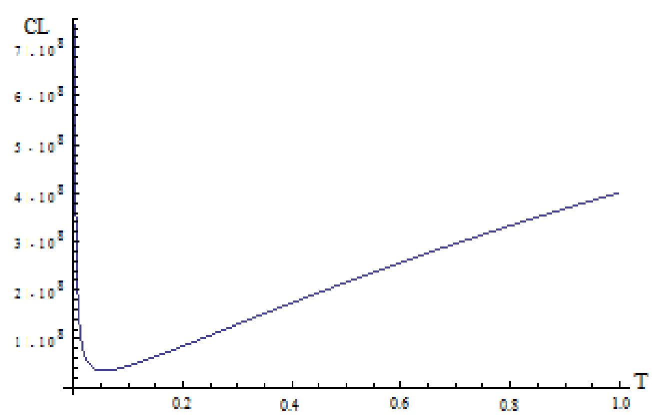


Figure 4: Inventory cost function at the case $M > T$

5.2. Sensitivity Analysis and Managerial Insights

In sensitivity analysis, the trade credit varies from M = 0 day (i.e. cash shopping) to M = 150, the expected lifetime of product is examined by changing from L = 2 years to L = 4 years, and the purchase cost per unit is considered to be changed from 10000 to 20000 Rs as shown in Table 2.

In analyzing the results of Table 2, it is clear that by reducing L, the inventory costs are concurrently reduced. This decline is predictable because by reducing L, the inventory will be held in fewer years; therefore, inventory costs will decrease as well. By analyzing M alone, no exact relationship is found between M and inventory cost. In some cases, in M < T, an inverse relationship can be observed and in other cases, in M > T, a direct relationship is observed. Also, in some cases, there is no trend between M and inventory cost. This means that M has an important role in achieving the optimal solution, which is the minimum inventory cost. Finally, by analyzing C_p alone, a direct relationship can be observed with inventory costs, since with greater C_p, the holding cost finds a greater role in constituting inventory costs in comparison with the other costs. However, changing C_p does not change the inventory costs as much as changing M.

Table 2: Inventory costs by changing M, L, and Cp

M < T					
C		$C_s = 1000$, $C_p = 10000$		$C_s = 2000$, $C_p = 20000$	
M	L	2	4	2	4
0	T	0.05119	0.04941	0.03629	0.03498
	C_L	3.09238×10^7	6.30284×10^7	4.33727×10^7	8.79639×10^7
	C_L (M)	∞	∞	∞	∞
60 days	T	0.10091	0.10822	0.09414	0.10242
	C_L	1.20381×10^7	4.3127×10^7	1.56768×10^7	7.08718×10^7
	C_L(M)	3.46762×10^7	6.9743×10^7	6.41529×10^7	1.29291×10^8
150 days	T	0.23097	0.24803	0.22811	0.24557
	C_L	1.61907×10^7	8.03952×10^7	2.86945×10^7	1.54109×10^8
	C_L(M)	7.96096×10^7	1.59034×10^8	1.57014×10^8	3.13867×10^8
M > T					
C		$C_s = 2000$, $C_p = 10000$ Rs		$C_s = 5000$, $C_p = 20000$ Rs	
M	L	2	4	2	4
0	T	0.10304	0.07583	0.07142	0.05342
	C_L	1.56873×10^7	4.05516×10^7	2.199×10^7	5.61513×10^7
	C_L (M)	∞	∞	∞	∞
60 days	T	0.09079	0.07314	0.06343	0.05155
	C_L	1.6021×10^7	4.03967×10^7	2.14607×10^7	5.49393×10^7
	C_L(M)	3.46762×10^7	6.9743×10^7	6.41529×10^7	1.29291×10^8
150 days	T	0.08022	0.06985	0.05633	0.04926
	C_L	1.06516×10^7	3.42361×10^7	9.38507×10^6	4.14163×10^7
	C_L(M)	7.96096×10^7	1.59034×10^8	1.57014×10^8	3.13867×10^8

Another analysis is also performed by changing the two effective parameters H (from 0.15 to 0.3) and I_e (from 0.05 to 0.14); where other parameters are fixed as shown in Table 3. The reason for this selection is due to the particular behavior of these two parameters in the objective function, which can affect the total inventory cost significantly. It is evident that by increasing H and decreasing Ie and Cp individually, an increase is observed in the inventory costs. The same results are also observed in the case M > T.

Table 3: Inventory costs by changing H, I_e, C

M < T							
		C_s =1000, C_p = 10000			C_s =2000, C_p = 20000		
H	I_e	0.05	0.08	0.14	0.05	0.08	0.14
0.15	T	0.12529	0.12249	0.11668	0.11941	0.11646	0.11032
	C_L	4.09021×10^7	3.77397×10^7	3.11905×10^7	6.85345×10^7	6.18952×10^7	4.80924×10^7
0.20	T	0.11525	0.11267	0.10731	0.10983	0.10711	0.10146
	C_L	5.24633×10^7	4.90295×10^7	4.19176×10^7	9.05176×10^7	8.33075×10^7	6.83165×10^7
0.3	T	0.10077	0.09851	0.09382	0.09602	0.09364	0.08869
	C_L	7.30546×10^7	6.91342×10^7	6.10136×10^7	1.29658×10^8	1.21425×10^8	1.04304×10^8

Managerial Insights

In today's competitive environment in pharmaceutical supply chains, despite the high presence of newcomers and developing new medicines that can lead to an early exit of the current medicines from medicines portfolio, the presence in the world of trade and the emphasis on the revenue that each product can make for the company would be impossible without managing the product life cycle and adopting appropriate strategies. Therefore, developing the optimization models by considering the probability of obsolescence of a product, especially in medical products is quite necessary. Also, it is recommended to create competitive advantages to attract the retailers for these types of products. One of these advantages is to consider the credit periods and set the delay in payment for retailers, which is considered in this study.

According to the results obtained from the implementation of the model, as expected, the inventory control policy considering delay in payment can impose fewer costs to the retailer (i.e. drugstores) because in this case, the drugstores can determine the optimal ordering cycle time and consequently optimal order quantity to make the highest profitable inventory values for them. This decision can be made by taking into account the financial costs arising from the remained items, which have paid their costs but not sold after credit period and the sold items that have not paid their cost during credit period up to their due date. Therefore, the drugstore retailers are advised to identify the medication and drugs, which are more exposed to obsolescence and then set the "delay in payment" contracts with their manufacturers and suppliers to protect themselves from the big loss resulted from outdating these medicines. Finally, the optimal ordering cycle time and optimal order quantity can also be obtained by the models, which are proposed in this study.

Conclusion

In this paper, an inventory control model for obsolescent items was introduced in the pharmaceutical retailing industry under the "delay in payment" policy given that the delay in payment offered by the suppliers to retailers makes retailers tend to order more. Accordingly, based on the risk of being obsolescence, more items would be sold, the obsolete items would be reduced, and as the result of decreasing the inventory costs, the retailer's profit would be increased.

Based on the presented model, in two cases of larger or smaller optimal ordering cycles than the time allowed for payment, the convexity of the model was proved by considering multiple lemma's and relations and also a simple algorithm for solving the model was developed. The numerical example showed that the increase in payment delays increases the ordering rate and, consequently, the delayed payment policy reduces the cost of depreciation.

In the presented model, after determining the inventory cost functions regarding obsolescence, holding and delay in payment costs, which are related to the pharmaceutical supply chain, the total cost function during the lifetime of items is introduced and the optimality is checked by convexity test through the second derivative in two cases of delay in payment in the model. Then a sensitivity analysis is performed on inventory costs using a triple (L, C, and M) and double (H, I_e) critical parameters of the model. Also, as the qualitative interpretation of the model, managerial insights are provided for drugstore retailers, regarding the numerical results obtained from sensitivity analyses. The numerical results showed that the length of delay in payment plays a critical role in determining the optimal solution, which is in fact the minimum total inventory cost. So, as the main result of this study, the use of credit period policy is advised to drugstore retailers, especially when the medicines are subject to sudden obsolescence; since this policy lets the retailers, to decrease their costs and simultaneously decrease the risk of encountering with the product obsolescence, by adjusting the interest earned and interest paid during the ordering cycle time. The use of mathematical models developed in this study will lead to optimal results.

As recommendations for future research, since quantity discount is among important marketing issues for selling medicine in Iran, discounts be implemented to the items exposed into obsolescence. Moreover, since the demand for a pharmacy product is not constant, the model can be considered with uncertain demands. Besides, the model can be developed with gradual obsolescence instead of sudden obsolescence. Moreover, trade credit can be changed with the volume of inventory. Furthermore, the model can be developed and solved with respect to the time-dependent prices and setting the different prices for obsolescent items. Finally, one can develop the coordination models for the closed-loop supply chain of obsolescent items.

Abbreviations:

(EOQ): Economic order quantity; (EPQ): Economic production quantity.

Competing interests:

The authors declare no competing interests.

Authors' Contributions

The authors contributed equally to this study.

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Appendix A

In the first case (M ≤ T) we have:

$$C_c = A + (H\,L\,C_p - C_s)*[TR - R\,L(1-e^{-\frac{T}{L}})] + C_p I_P\left(\frac{RM^2}{2} - MTR + \frac{RT^2}{2}\right) - C_p I_e\,R\left[L - e^{-\frac{M}{L}}(L+M)\right] \quad \text{(A1)}$$

In order to simplify the first and second derivatives, the following relationships are replaced:

$$F_1 = A + C_p I_p \frac{RM^2}{2} - (H\,L\,C_p - C_s)R\,L - C_p\,I_e\;R[L - e^{-\frac{M}{L}}(L+M)] \quad \text{(A2)}$$

$$F_2 = (H\,L\,C_p - C_s)R - M\,R\,C_p\,I_p \quad \text{(A3)}$$

$$F_3 = \frac{R\,C_p\,I_p}{2} \quad \text{(A4)}$$

$$F_4 = (H\,L\,C_p - C_s)R\,L \quad \text{(A5)}$$

Then we will have:

$$C_c = F_1 + F_2 T + F_3 T^2 + F_4 e^{-\frac{T}{L}} \quad \text{(A6)}$$

Therefore, the total inventory costs during the expected life time of the product would be:

$$C_L = \frac{F_1 + F_2 T + F_3 T^2 + F_4 e^{-\frac{T}{L}}}{1-e^{-\frac{T}{L}}} \quad \text{(A7)}$$

So, the first derivative of the function is:

$$\frac{\partial C_L}{\partial T} = \frac{F_2 + 2F_3 T - \frac{F_4 e^{-\frac{T}{L}}}{L}}{1-e^{-\frac{T}{L}}} - \frac{e^{-\frac{T}{L}}(F_1 + F_2 T + F_3 T^2 + F_4 e^{-\frac{T}{L}})}{L\left(1-e^{-\frac{T}{L}}\right)^2} \quad \text{(A8)}$$

And the second derivative of the function is:

$$\frac{\partial^2 C_L}{\partial T^2} = \frac{2F_3 + \frac{F_4 e^{-\frac{T}{L}}}{L^2}}{1-e^{-\frac{T}{L}}} - \frac{2e^{-\frac{T}{L}}\left(F_2 + 2F_3 T - \frac{F_4 e^{-\frac{T}{L}}}{L}\right)}{L\left(1-e^{-\frac{T}{L}}\right)^2} + (F_1 + F_2 T + F_3 T^2 + F_4 e^{-\frac{T}{L}})\left(\frac{2e^{-\frac{2T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^3} + \frac{e^{-\frac{T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^2}\right) \quad \text{(A9)}$$

In other word, we will have:

$$\frac{\partial^2 C_L}{\partial T^2} = \frac{2F_3 + \frac{F_4\,e^{-\frac{T}{L}}}{L^2}}{1-e^{-\frac{T}{L}}} + F_1 * \left[\frac{2e^{-\frac{2T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^3} + \frac{e^{-\frac{T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^2}\right] + F_2 * \frac{e^{-\frac{T}{L}}}{L\left(1-e^{-\frac{T}{L}}\right)^2} * \left[\frac{2\,T\,e^{-\frac{T}{L}}}{L\left(1-e^{-\frac{T}{L}}\right)} + \frac{T}{L} - 2\right] + F_3 * \frac{Te^{-\frac{T}{L}}}{L\left(1-e^{-\frac{T}{L}}\right)^2} * \left[\frac{2\,T\,e^{-\frac{T}{L}}}{L\left(1-e^{-\frac{T}{L}}\right)} + \frac{T}{L} - 4\right] + F_4 * \frac{e^{-2\frac{T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^2} * \left[\frac{2\;e^{-\frac{T}{L}}}{1-e^{-\frac{T}{L}}} + 3\right] \quad \text{(A10)}$$

Since $F_3 \geq 0$ and also we know that $(1 - e^{-\frac{T}{L}}) > 0$ we can conclude that the objective function is convex ($\frac{\partial^2 C_L}{\partial T^2} \geq 0$) with respect to the following relations:

$$A + C_p I_e \frac{RM^2}{2} + C_p\, I_e\;\; R\, [e^{-\frac{M}{L}}(L + M)] > R\, L\, [(H\, L + I_e) C_p \text{-}\, C_s] \quad \text{(A11)}$$

$$H\, L\, C_p > C_s + M\, C_p\, I_p \quad \text{(A12)}$$

$$\frac{2\, T\, e^{-\frac{T}{L}}}{\mathrm{L}\left(1 - \mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{T}{\mathrm{L}} > 4 \quad \text{(A13)}$$

Appendix B

In the second case (M > T) we have:

$$C_c = A + \left(H\, L\, C_p - C_s\right) \;*\; \left[TR - R\, L\left(1 - e^{-\frac{T}{L}}\right)\right] - C_p\, I_e\, R\, [L - e^{-\frac{T}{L}}(L + T)] - C_p\, I_e\;\; TR\, \left[\left(-e^{-\frac{M}{L}}(L + M - T) + L\, e^{-\frac{T}{L}}\right)\right] \quad \text{(B1)}$$

In order to simplify the first and second derivatives, the following relationships are replaced.

$$G_1 = A - C_p\, I_e\, R\, L - \;(H\, L\, C_p \text{ - } C_s) R\, L \quad \text{(B2)}$$

$$G_2 = \;(H\;\; L\, C_p\text{-}\, C_s) R + \; C_p\, I_e\, R\, \left[e^{-\frac{M}{L}}(L + M)\right] \quad \text{(B3)}$$

$$G_3 = -\, C_p\, I_e\, R\, e^{-\frac{M}{L}} \quad \text{(B4)}$$

$$G_4 = \left(H\, L\, C_p - C_s\right) R\, L + C_p\, I_e\, R\, L \quad \text{(B5)}$$

$$G_5 = C_p\, I_e\, R \text{ - } C_p\, I_e\, R\, L \quad \text{(B6)}$$

At result, we will have:

$$C_c = G_1 + G_2\;\mathrm{T} + G_3\;\mathrm{T}^2 + G_4\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}} + G_5\;\mathrm{T}\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}} \quad \text{(B7)}$$

Therefore, the total inventory costs during the expected life time of the product would be:

$$\mathrm{C_L} = \frac{G_1 + G_2\;\mathrm{T} + G_3\;\mathrm{T}^2 + G_4\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}} + G_5\;\mathrm{T}\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{1 - \mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}} \quad \text{(B8)}$$

So, the first derivative of the function is:

$$\frac{\partial C_L}{\partial \mathrm{T}} = \frac{G_2 + 2\, G_3\;\mathrm{T} - \frac{(G_4 + G_5\;\mathrm{T})\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}} + G_5\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{1 - \mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}} - \frac{\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}(G_1 + G_2\;\mathrm{T} + G_3\;\mathrm{T}^2 + G_4\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}} + G_5\;\mathrm{T}\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}})}{L\left(1 - e^{-\frac{T}{L}}\right)^2} \quad \text{(B9)}$$

And the second derivative of the function is:

$$\frac{\partial^2 C_L}{\partial \mathrm{T}^2} = \frac{2\, G_3 + \frac{(G_4 + G_5\;\mathrm{T})\; e^{-\frac{T}{L}}}{L^2} - \frac{2\, G_5 e^{-\frac{T}{L}}}{L}}{1 - e^{-\frac{T}{L}}} - \frac{2 e^{-\frac{T}{L}}(G_2 + 2\, G_3\;\mathrm{T} - \frac{(G_4 + G_5\;\mathrm{T})\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}} + G_5\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}})}{L\left(1 - e^{-\frac{T}{L}}\right)^2} + (G_1 + G_2\;\mathrm{T} + G_3\;\mathrm{T}^2 + G_4\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}} + G_5\;\mathrm{T}\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}})\left(\frac{2 e^{-\frac{2T}{L}}}{L^2\left(1 - e^{-\frac{T}{L}}\right)^3} + \frac{e^{-\frac{T}{L}}}{L^2\left(1 - e^{-\frac{T}{L}}\right)^2}\right) \quad \text{(B10)}$$

$$\frac{\partial^2 C_L}{\partial \mathrm{T}^2} = G_1 * \left[\frac{2 e^{-\frac{2T}{L}}}{L^2\left(1 - e^{-\frac{T}{L}}\right)^3} + \frac{e^{-\frac{T}{L}}}{L^2\left(1 - e^{-\frac{T}{L}}\right)^2}\right] + G_2 * \frac{\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{L\left(1 - e^{-\frac{T}{L}}\right)^2} * \left[\frac{2\;\mathrm{T}\;\mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1 - \mathrm{e}^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{\mathrm{T}}{\mathrm{L}} \;\; - \;\; 2\right] \;\; + \quad \text{(B11)}$$

$$G_3 * \frac{\mathrm{T}e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)^2} * \left[\frac{2\,\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)}{\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}} + \frac{2\,\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{\mathrm{T}}{\mathrm{L}} - 4\right] + G_4 * \left[\frac{e^{-\frac{T}{L}}}{L^2\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{3e^{-\frac{2T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^2} + \frac{2e^{-\frac{3T}{L}}}{L^2\left(1-e^{-\frac{T}{L}}\right)^3}\right] + G_5 * \frac{e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{L\left(1-e^{-\frac{T}{L}}\right)^2} * \left[-2 + \frac{\mathrm{T}}{\mathrm{L}} + \frac{2\,\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)}\right]$$

Considering:

$$\frac{Ln(L+M) - Ln(L)}{M} \cong \frac{1}{L} \Longrightarrow \frac{L}{L+M} \cong e^{-\frac{M}{L}} \qquad \text{(B12)}$$

We will have:

$$G_2 + G_5 \cong (H\ L\ C_p \text{-}\ C_s)R + C_p\, I_e\, R \qquad \text{(B13)}$$

Therefore, with respect to the following relations, the objective function would be convex.

$$A > [(H\ L + I_e)\ C_p\ \text{-}\ C_s]\ R\ L \qquad \text{(B14)}$$

$$H\ L\ C_p > C_s \qquad \text{(B15)}$$

$$\frac{2\,\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{\mathrm{T}}{\mathrm{L}} > 2 \qquad \text{(B16)}$$

$$\frac{2\,\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)}{\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}} + \frac{2\,\mathrm{T}\,e^{-\frac{\mathrm{T}}{\mathrm{L}}}}{\mathrm{L}\left(1-e^{-\frac{\mathrm{T}}{\mathrm{L}}}\right)} + \frac{\mathrm{T}}{\mathrm{L}} < 4 \qquad \text{(B17)}$$