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Cost efficiency analysis of network DEA models: the case of Mashhad hospitals Running title: Cost efficiency analysis of network DEA models

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Abstract
Background and objectives: This paper presents cost efficiency analysis of network models with parallel structure. Through measuring cost efficiency of this type of systems, conventional cost efficiency model considers the system as a whole and neglects the operations of the internal processes.
Methods: As a theoretical contribution, the parallel cost efficiency model was proposed to utilize the operation of internal processes for measuring cost efficiency of the system. This study proposed a cost efficiency decomposition of parallel systems, where each process utilized external inputs to produce external outputs. In addition to the theoretical aspect, as an empirical contribution, we implemented the proposed model for cost efficiency analysis of hospitals in Mashhad for two levels, i.e. hospital level and ward level.
Finding: Ignoring organizational structure of hospitals and considering them as black boxes, the average technical efficiency of hospitals was found to be 0.6 for. Then considering the internal structure of hospitals, the technical efficiency values of hospitals changed into 0.1. Taking cost efficiency measures into account, it was found that the average cost efficiency of hospital was low, namely, about 0.32 that is attributed to hospital's system; however, the average cost efficiency of wards was much higher, i.e. about 0.38, though it was still low. Among different wards, ICU ward ranked the first level in terms of cost efficiency, followed by emergency ward and operation ward ranked the second and third, respectively.
Conclusion: Internal structure of hospitals, as a complex organization, should be taken into account in efficiency analysis to have a better insight regarding performance. It is worth mentioning that in hospitals in Mashhad more attention must be paid to cost minimization and cost management, specifically in emergency wards and operation wards.
***Keywords**:* Data Envelopment Analysis, Hospital Cost Efficiency, Ward Cost Efficiency, Network Cost Analysis

Background and objectives:

Efficiency analysis of decision making units (DMUs) is a crucial task that involves internal structure of production system and external links between other production units; it may affect the performance of a unit under evaluation in a market. Data envelopment analysis (DEA) technique is one of the promising methods to evaluate the performance of a given organization[1]. Different versions of this technique for different purposes have been proposed in the literatures so far. Incorporating cost information could be considered as one of the developments and an important factor in representing the process of efficiency analysis in a market. DEA technique is an alternative for measuring cost efficiency (CE) of production units.

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In a different line of research, network DEA models are proposed to reveal internal structure of production units which looks like a black box. A summary of the review of related researches are presented below.

Review of related research

For the first time, the idea of cost efficiency has introduced in study of Farrell[2]. Farrell's idea was further developed and a linear programming (LP) model for calculating cost efficiency of system was formulated in research of Färe and Grosskopf[3]. The shortages of cost and allocative efficiency measures considering price differences between DMU is explained in Tone`s research[4]. The law of one price using weight restriction DEA model in the multiplier form is also studied and investigated[5]. Production cost efficiency in the presence of ordinal data is investigated in research of Jahanshahloo et al[6]. All the mentioned researches have considered the implementation of weight restriction on multiplier DEA problem[7].

Uniqueness of the solution is shown in research of Kuosmanen[5] in which two bi-level models are proposed for finding a unique solution in the presence of the law of one price. However, price uncertainty exist in the production units, hence, dealing with cost efficiency problem is studied in research of Camanho and Dyson[8]. In all the reviewed papers production units are assumed to have a simple and straightforward input-output link. Problems with network structure could be easily found in real world. Efficiency analysis of this type of problem is performed using network DEA (NDEA) models that can identify reasons of efficiency and inefficiency in subunits and the whole system. So far, several studies have investigated NDEA. In order to measure system efficiency, the operations of internal processes was taken into account as the first step[9]. Some references including utilized NDEA singularly. Parallel structure is one of basic structures of a network system[10-13]. Hierarchical structures are described in research of Castelli et al[14] where each system is formed by consecutive stages of parallel processes. DMUs assumed as black boxes by Sexton et al[15] that are consisted of sub-DMUs; some of these sub-DMUs consume resources produced by other sub-DMUs. In order to decompose inefficiency slack of the system to the sum of those processes in parallel systems, a slack-based measure (SBM) model was developed in Tone and Tsutsui[16]. A comprehensive categorized overview of models and methods developed for different multi-process systems is presented in research of Castelli et al[17]. A parallel DEA model used to measure system efficiency that could be decomposed into efficiencies of the processes is described in Kao`s resrarch[18]. The dynamic effects of productions in efficiency analysis of network structured DEA models is studied in Chen`s study[19] and proposed new measure in this context. In addition to internal relationships, Chen investigated interconnections between classical and new measures[19]. In a similar study investigating dynamic structure of network models, Kou et al proposed a framework for efficiency measurement of multi period and multi division production systems[20]. A network DEA and its related weight decomposition in an additive form is proposed in research of Cook et al[21]. A review paper on different developments of network DEA models until 2014 is presented in Kao`s reseach[22]. A book chapter discussing pros and cons of network DEA models is presented in research of Chen et al[23] that considers two general sets of network DEA models, one based on envelopment form and the other one based on multiplier form. It is concluded that an envelopment-based network DEA model should be used to determine the inefficiency of frontier projection.

The healthcare sector in general and hospitals in particular are among the main areas for the application of DEA[24]. New diagnostic and therapeutic methods are implemented to combat the rising rate of chronic diseases, the increasing demand for health services, and the subsequent medical errors[25]. Thus, hospitals are an essential component of health systems, and the most costly parts as well. They account for 50–80% of total health expenditures[26]. On the other hand, the measurement of efficiency and productivity is crucial for hospitals because it allows them to compare the performance of their own organization with that of other hospitals[27]. As Ahmed et al pointed out, studies on efficiency are important for informed decision making to improve the performance of hospitals and reduce expenditures[28]. It is important to reduce the consumption of excessive resources for providing healthcare services in all healthcare systems. Efficiency in production always results in better allocation of resources and increases the opportunity to serve more beneficiaries. Nowadays, in most developing countries, 5% to 10% of budget is dedicated to the healthcare sector. In addition, 50% to 80% of the budget of health sector is dedicated to hospitals. In Iran, about 40% of the total public health care budget is allocated to hospital care[29]. Given the resource scarcity in health sector, it is necessary to increase the efficiency of healthcare organizations[30]. Hence, paying attention to efficiency measurement is considered as one of the main pillars required for improving the level of efficiency in hospitals[29]. Until recently, measures such as per capita costs of hospital care in a hospital district, or average cost of patient day, discharge or visit were the only available evidences for making managerial decisions. Obviously, this type of measure provides little insight into the complex issue of productive efficiency and presents insufficient data for operative planning. As stated by Hernandez and Sebastian[31], inputs are uniform and low and health outcomes could increase when making efforts to achieve better health promotion in primary and secondary healthcare sector. They also point out that in many cases the needs for healthcare services are poorly met. Stefko et al performed a window analysis to evaluate healthcare technical efficiency in individual regions[32]. They found high degree of efficiency for those regions with low values of the variables over time[32]. Ali et al investigated the technical efficiency of hospitals in Eastern Ethiopia in 2007-2013[33]. They argued that hospitals consumed a larger proportion of total public health budget. Even though the percentage varies from country to country, the efficiency of hospitals needs to receive enough attention as the budget they consume is enormous. It is recognized that improved efficiency is one of the main goals of health systems. In Karahan`s research[34], efficiency status of nine Turkish hospitals providing similar services was analyzed using DEA approach and required target setting was suggested for inefficient hospitals. An efficiency analysis of Saudi Arabian’s public hospitals was performed by Helal[35] to assess services provided by each hospital. As they reported, the average production efficiency of the services provided (internally) by districts of the Kingdom of Saudi Arabia was 94.7%, and the average external production efficiency for such services was 95.4%. A systematic review of various techniques for efficiency measurement of health care organizations in Iran is presented by Jaafaripooyan et al[36]. They found that 73 out of 122 studies employed DEA technique for measuring the efficiency of health care organizations and 23 studies utilized hybrid models including DEA. Ravaghi et al conducted a systematic review of hospital inefficiency in the Eastern Mediterranean region and concluded that inefficiencies were considered to be attributed to excess workforce, excess beds, inappropriate hospital size, inappropriate workforce composition, lack of workforce motivation, and inefficient use of health system inputs. It is suggested that health policymakers and managers use this evidence to develop appropriate strategies for reducing hospital inefficiency[37]. A decline in hospital efficiency has been observed worldwide. In a global report by the World Health Organization (WHO) published in 2010, 10 sources of hospital inefficiency were identified: (1) underuse or overpricing of generic drugs; (2) use of substandard or counterfeit drugs; (3) inappropriate and ineffective drug use; (4) overuse or oversupply of equipment, investigations, and procedures; (5) inappropriate or costly workforce mix, and recruiting unmotivated workers; (6) inappropriate hospital admissions or length of stay; (7) inappropriate hospital size (low use of infrastructure); (8) medical errors and suboptimal quality of care; (9) waste, corruption, and fraud; and (10) inefficient mix or inappropriate level of strategies (37). Evaluating hospital efficiency is a process used to optimize resource utilization and allocation, which is vital because hospitals spent the largest financial costs in a health system. To limit the excessive use of hospital resources, especially in avoidable processes, it is important to identify the sources of hospital inefficiencies and adopt measures to reduce and eliminate them[37].

Our objective and contribution

As the empirical contribution of our study, the main objective of this paper is to present the results of cost efficiency analysis in hospitals in Mashhad. To this end, we developed a proper cost efficiency model, which considered the parallel structure of hospitals and resulted in the theoretical contribution of this study.

None of the reviewed papers presented in the previous section have considered price information in network structure in the procedure of efficiency analysis. To the best of our knowledge, there is only one recent paper which addressed the issue of cost efficiency in the network structure[38]. It proposed network DEA model for measuring cost efficiency of DMUs in a static and dynamic framework. The law of one price for an activity is a main factor which is ignored in research of Seyedboveir et al[38]. This is more crucial when we perform an empirical analysis that needs to consider this law. However, the current study utilizes parallel DEA technique and cost efficiency model to provide parallel cost efficiency (PCE) model. This model assesses PCE by taking into account the law of one price. Based on the law of one price for an activity, the price of similar services, activities, or products are the same in a production system under evaluation. This law is applicable in our research, because all the hospitals are public and the price of similar inputs are the same in our case study.

Most production systems in the real word have complex internal structure. Network DEA models considers this structure in the procedure of efficiency analysis. Another important fact that usually exists in all production systems is the limitation of resources. Almost all production systems struggle with the financial and budgeting constraints at different levels. It highlights the importance of cost analysis, not only in complex production systems but also in those systems that may have a simple and straightforward production structure. Cost analysis methodology already exist in the literature to be applied for simple structured systems. We propose the theoretical

foundation of cost efficiency analysis for those production systems with more complex internal structure rather than simple and straightforward production structure.
In fact, while conventional cost efficiency model considers systems as a whole, the PCE model proposed in this study estimates the cost efficiency of a system using process cost efficiency. Moreover, we propose relative decompositions to find cost efficiency of a process as well. As a result, the decision maker will be able to identify cost inefficient processes and improve them. A theoretical and empirical comparison between our model and other existing models is provided in the literature.

The rest of this paper is presented as follows. The parallel DEA model used to calculate system and process efficiencies is explained in section 2. The conventional cost efficiency model used to measure the system cost efficiency is presented in section 3. The extension of conventional cost efficiency model to parallel cost efficiency (PCE) model is described in section 4. An application of the proposed model to hospitals in Mashhad is presented in section 5. At the end, a brief conclusion of this study is provided in section 6.

Methods:

Parallel DEA model

Parallel structure is one of the network structures which corresponds to systems constructed by a number of parallel processes, as shown in Fig. 1. In this type of systems, in each process p, $p = 1,\dots,q$ converts inputs $X_i^{(p)}, i \in I^{(p)}$, into outputs $Y_r^{(p)}, r \in O^{(p)}$. The sums of the inputs $X_i^{(p)}$ and outputs $Y_r^{(p)}$ for all q processes are the system's inputs $\sum_{p=1}^{q} X_{ij}^{(p)} = X_{ij}$ and system's outputs $\sum_{p=1}^{q} Y_{rj}^{(p)} = Y_{rj}$ for each DMU_j. The following model is proposed to calculate system efficiency in a parallel structure (39):

$$
\begin{aligned}
& min. \quad \theta \\
& s.t: \quad \sum_{j=1}^{n}\sum_{p=1}^{q} \lambda_j^{(p)} X_{ij}^{(p)} \le \theta X_i, \qquad i = 1,\dots,m, \\
& \sum_{j=1}^{n}\sum_{p=1}^{q} \lambda_j^{(p)} Y_{rj}^{(p)} \ge Y_{rk}, \qquad r = 1,\dots,s, \\
& \lambda_j^{(p)} \ge 0\,, \qquad p = 1,\dots,q,\ j = 1,\dots,n. \qquad (1)
\end{aligned}
$$

The above model find the efficiency of whole system. If $\theta = 1$ in the optimal case then the hole system if efficient otherwise it is inefficient. Note that process efficiencies are not determined in model (1). The presented model considers constant returns to scale. Assuming that the returns to scale are allowed to be variable, then an intensity constraint $\sum_{j=1}^{n}\sum_{p=1}^{q}\lambda_j^{(p)} = 1$ will be added to the model. In fact, the proposed models consider constant returns to scale and are input orientated. They can be adapted in output orientation or having variable returns to scale property with some minor notation adaption, if required.
Model (1) has a dual form, as presented below:

$$
\begin{aligned}
& max\ E_k. \quad \sum_{r=1}^{s} u_{rk} Y_{rk} \\
& s.t: \quad \sum_{i=1}^{m} v_{ik} X_{ik} = 1, \\
& \sum_{r \in O^{(p)}} u_{rk} Y_{rj}^{(p)} - \sum_{i \in I^{(p)}} v_{ik} X_{ij}^{(p)} \le 0, p = 1,\dots,q, \\
& j = 1,\dots,n, \\
& u_{rk}, v_{ik} \ge \varepsilon, r = 1,\dots,s, i = 1,\dots,m. \qquad (2)
\end{aligned}
$$

This model is trying to find the maximum producible output of the whole system with respect to the current resources. For more description of dual relationship between model (1) and model (2) one may see (40) for instance. This model is called parallel DEA model and measures system efficiency and process efficiency in a unified framework. This parallel model indicates that system efficiency is calculated as the main objective and process efficiencies as its components.

Solving the above model (2), we can calculate system and its process efficiencies, as presented below:

$$E_k = \frac{\sum_{r=1}^{s} u_{rk}^{*} Y_{rk}}{\sum_{i=1}^{m} v_{ik}^{*} X_{ik}} = \sum_{r=1}^{s} u_{rk}^{*} Y_{rk} \quad (3)$$

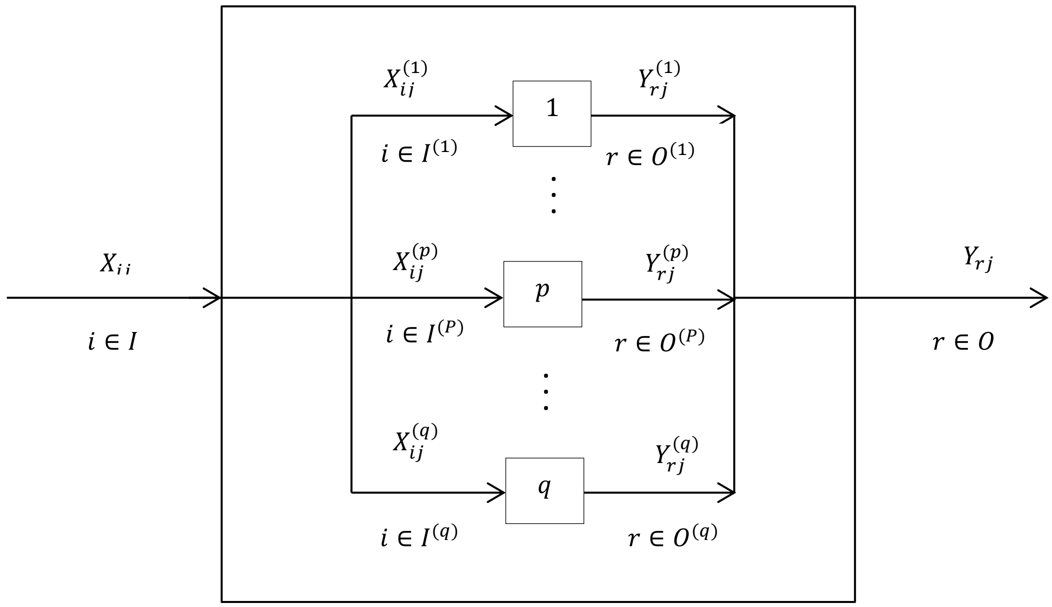

Figure.1. The parallel production system, where DMU_k has q production units.

$$E_k^{(p)} = \frac{\sum_{r \in O^{(p)}} u_{rk}^{*} Y_{rk}^{(p)}}{\sum_{i \in I^{(P)}} v_{ik}^{*} X_{ik}^{(p)}} \quad , p = 1, \dots, q \quad (4)$$

As a matter of fact, regarding to the system efficiency gauged by model (2) and relative optimal weights determine the process efficiencies. It means that for calculating process efficiencies, we use the same optimal weight that gives us the system efficiencies. It is worth noting that the given bundle weights in equation (3) and equation (4) have the same logic, one for efficiency of the whole system and the other for efficiency of processes. The mentioned logic comes from one of the main definitions of relative efficiency, i.e. the optimal weighted output divided by the optimal weighted input for the DMU (system in the network structure) or a sub-DMU (process in the network structure). The weight associated with process p can be defined as follows:

$$w^{(p)} = \frac{\sum_{i \in I^{(P)}} v_{ik}^{*} X_{ik}^{(p)}}{\sum_{i=1}^{m} v_{ik}^{*} X_{ik}}$$

As a result, the average of the q process efficiencies weighted by $w^{(p)}$ is:

$$\sum_{p=1}^{q} w^{(p)} E_k^{(P)} = \sum_{p=1}^{q} \left[\left(\frac{\sum_{i \in I^{(P)}} v_{ik}^{*} X_{ik}^{(p)}}{\sum_{i=1}^{m} v_{ik}^{*} \mathrm{X}_{ik}} \right) \left(\frac{\sum_{r \in O^{(p)}} u_{rk}^{*} Y_{rk}^{(p)}}{\sum_{i \in I^{(P)}} v_{ik}^{*} X_{ik}^{(p)}} \right) \right] = \sum_{p=1}^{q} \left(\frac{\sum_{r \in O^{(p)}} u_{rk}^{*} Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_{ik}^{*} \mathrm{X}_{ik}} \right) = \frac{\sum_{r=1}^{s} u_{rk}^{*} \mathrm{Y}_{rk}}{\sum_{i=1}^{m} v_{ik}^{*} \mathrm{X}_{ik}} \quad (5).$$

In fact, above equation finds the relationship between the whole system and its processes. The system efficiency is weighted sum of process efficiency and the relative weight of the p-th process ($p = 1,2,\dots,q$) is defined as $w^{(p)} = \frac{\sum_{i \in I^{(P)}} v_{ik}^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_{ik}^* X_{ik}}$. In other words, the associated weight of $w^{(p)}$ determines the efficiency share of p-th process for the system efficiency.

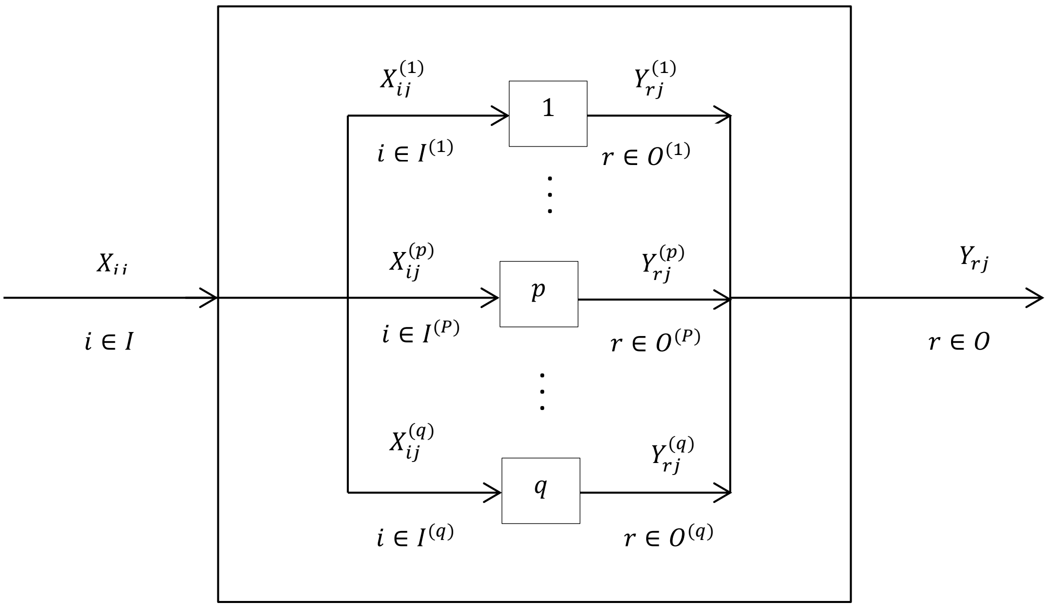

Figure.1. The parallel production system, where DMU_k has q production units.

The system efficiency in a parallel structure will represent the aggregate performance of all of its processes when the operation of each process is taken into account.

Conventional cost efficiency model

In DEA, $T_{CCR} = \{(X,Y): X \geq \sum_{j=1}^{n} X_j\lambda_j, Y \leq \sum_{j=1}^{n} Y_j\lambda_j, \lambda_j \geq 0, j = 1,\dots,n\}$ represents production possibility set (PPS) under the assumption of constant returns to scale. When there are n congruent DMU ($DMU_j;\ j = 1,\dots,n$), the DMU_jconverts m inputs $X_{ij}, i = 1,\dots,m$ into s outputs $Y_{rj}, r = 1,\dots,s$. In order to calculate system cost efficiency based on the estimated production possibility set T_{CCR}, a model is proposed as presented below (3):

$$\begin{aligned}
min.\ & \sum_{i=1}^{m} c_i X_i \\
s.t:\ & \sum_{j=1}^{n} \lambda_j X_{ij} \leq X_i, \quad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rk}, \quad r = 1,\dots,s \\
& \lambda_j \geq 0, \quad j = 1,\dots,n \qquad (6)
\end{aligned}$$

Transferring to the dual space, the following model is obtained as a dual of the model (6):

$$\begin{aligned}
max.\ & \sum_{r=1}^{s} u_{rk} Y_{rk} \\
s.t:\ & \sum_{r=1}^{s} u_{rk} Y_{rj} - \sum_{i=1}^{m} v_{ik} X_{ij} \leq 0,\ j = 1,\dots,n, \\
& v_{ik} - c_i \leq 0, i = 1,\dots,m \\
& u_{rk}, v_{ik} \geq \varepsilon, r = 1,\dots,s, i = 1,\dots,m. \quad (7)
\end{aligned}$$

Variables implemented in the given model are described as follows: X_{ij} is the amount of ith input utilized by jthDMU; $Y_{rj}$is the amount of rth output produced by jth DMU; $v_{ik}$and u_{rk} are weights for the ith input and rth output of kth DMU respectively; k $(k = 1, \dots, n)$ presents DMU under the evaluation; and c_i indicates price of ith input. Solving the multiplier form of the model (7) yields the cost efficiency of kth DMU, that is, $CE_k = \frac{\sum_{r=1}^{s} u_{rk}^* Y_{rk}}{\sum_{i=1}^{m} v_{ik}^* X_{ik}}$.

Proposed parallel cost efficiency (PCE) model

The existing cost efficiency model utilized for measuring cost efficiency of system is limited to a black-box model. However, in real-world problems, some systems have parallel structure. In this section, cost efficiency model for parallel systems is explained. The results of this model indicate that through adopting a network perspective, it is possible to calculate cost efficiency of all the processes and of course the whole system. Thus, the PCE model in the original production possibility set for a parallel system can be formulated as follows:

$$
\begin{aligned}
min. \quad & \sum_{i=1}^{m} c_i X_i \\
s.t: \quad & \sum_{j=1}^{n} \sum_{p=1}^{q} \lambda_j^{(p)} X_{ij}^{(p)} \le X_i, \qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \sum_{p=1}^{q} \lambda_j^{(p)} Y_{rj}^{(p)} \ge Y_{rk}, \qquad r = 1, \dots, s, \\
& \lambda_j^{(p)} \ge 0\,, \quad p = 1, \dots, q,\ j = 1, \dots, n. \qquad (8)
\end{aligned}
$$

The dual form of the above mentioned model is as follows:

$$
\begin{aligned}
max. \quad & \sum_{r=1}^{s} u_{rk} Y_{rk} \\
s.t: \quad & \sum_{r \in O^{(p)}} u_{rk} Y_{rj}^{(p)} - \sum_{i \in I^{(p)}} v_{ik} X_{ij}^{(p)} \le 0, \\
& p = 1, \dots, q\,, j = 1, \dots, n, \\
& v_{ik} - c_i \le 0, \qquad i = 1, \dots, m, \\
& u_{rk}\,, v_{ik} \ge \varepsilon\,, r = 1, \dots, s, i = 1, \dots, m. \qquad (9)
\end{aligned}
$$

We use the dual model (9) to calculate cost efficiencies of the system and its process. To describe parallel cost efficiency in (9), it is assumed that the performance of n DMUs $(DMU_j;\ j = 1, \dots, n)$ with q production processes is measured. The p production process utilizes m inputs $X_{ij}^{(p)}, i = 1, \dots, m, p = 1, \dots, q$ to produce s outputs $Y_{rj}^{(p)}, r = 1, \dots, s, p = 1, \dots, q$; in addition, k $(k = 1, \dots, n)$ presents the DMU under the evaluation. Moreover, c_i indicates price of ith input. The model is applied separately for each DMU.

When (u_{rk}^*, v_{ik}^*) is an optimal solution of the model (9), the system and process cost efficiencies can be calculated as:

$$CE_k^N = \frac{\sum_{r=1}^{s} u_{rk}^* Y_{rk}}{\sum_{i=1}^{m} v_{ik}^* X_{ik}} \qquad (10)$$

$$CE_k^{(p)} = \frac{\sum_{r \in O^{(p)}} u_{rk}^* Y_{rk}^{(p)}}{\sum_{i \in I^{(p)}} v_{ik}^* X_{ik}^{(p)}}\,, p = 1, \dots, q. \qquad (11)$$

If we define the weight associated with process p as the aggregate input consumed by this process which is consumed by all q processes, $w^{(p)} = \frac{\sum_{i \in I^{(p)}} v_{ik}^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_{ik}^* X_{ik}}$,then the weighted average of q process cost efficiencies is system cost efficiency described by the following proposition.

Proposition 1. Ssystem cost efficiency of q processes with a parallel structure is a weighted average of q process cost efficiencies.

Proof. See appendix A.

Proposition 2. The optimal value of PCE model (9) is not larger than the optimal value of CE model (7); hence, cost efficiency score of a DMU in a network (CE_k^N) is less than or equal to its cost efficiency calculated using conventional cost efficiency model(CE_k).

Proof. See appendix A.

Now we compare our PCE model (8) with statistic network cost efficiency model (SNCE) of (38). Neglecting intermediate product and one set of input price, (SNCE) of (38) is as follows:

$$min. \quad \sum_{i=1}^{m} c_i X_i$$

$$s.t: \quad \sum_{j=1}^{n} \lambda_j^{(p)} X_{ij}^{(p)} \leq X_i,$$

$$i = 1, \dots, m_p, p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j^{(p)} Y_{rj}^{(p)} \geq Y_{rk},$$

$$r = 1, \dots, s, p = 1, \dots, q$$

$$\lambda_j^{(p)} \geq 0\,, \quad p = 1, \dots, q,$$

$$j = 1, \dots, n. \qquad (13)$$

Proposition 3. The optimal value of PCE model (8) is not larger than the optimal value of SNCE of (13).

Proof. See appendix A.

Results:

Cost and technical efficiency analysis in hospitals in Mashhad

In addition to classic technical and network analysis, in this section we perform a cost efficiency analysis in public hospitals in Mashhad. Each process (ward) in the parallel system uses the same number of inputs to produce the same number of outputs and all DMUs (hospitals) have the same number of processes. Although hospital have a complex internal structure and interactions with other related organizations, because of data availability which will be reported later, we ignore interactions between wards and we consider them as parallel service systems. In this case each unit has some processes operating independently with a unique manager in the hospital. We analyzed eleven public hospitals in Mashhad in terms of cost efficiency. Three wards are considered in each hospital. For each hospital and each ward in this study, number of active beds (X_1), number of physicians (X_2), and number of nurses(X_3) are considered as the inputs of the presented model, whereas its output is the total number of patients admitted to those places. Table 1 provides a statistical summary of the data. Given the number of DMUs (hospitals) and sub-DMUs (wards) (eleven DMUs and 33 sub-DMUs), we do not face any problem in terms of discriminating power in our analysis. Our results, reported in Tables 2 and 3, also support this fact.

Table 1: descriptive statistics for 33 Mashhad hospitals in 2017

Ward # 1

		Number of active bed	Number of physician	Number of nurse	Total of admitted patients
N	Valid	11	11	11	11
	Missing	0	0	0	0
Mean		121.82	34.73	108.82	6640.91
Std. Deviation		103.128	31.145	98.085	5357.013
Minimum		32	9	28	1333
Maximum		319	95	299	17699
Sum		1340	382	1197	73050

Ward # 2

		Number of active bed	Number of physician	Number of nurse	Total of admitted patients
N	Valid	11	11	11	11
	Missing	0	0	0	0
Mean		45.36	2.27	58.27	5679.09
Std. Deviation		60.422	1.489	79.554	6227.612
Minimum		3	1	4	787
Maximum		169	5	239	19243
Sum		499	25	641	62470

Ward # 3

		Number of active bed	Number of physician	Number of nurse	Total of admitted patients
N	Valid	11	11	11	11
	Missing	0	0	0	0
Mean		35.09	1.73	68.45	858.55
Std. Deviation		38.754	1.489	82.331	892.862
Minimum		4	1	6	111
Maximum		114	6	235	2836
Sum		386	19	753	9444

Like classical DEA model, sensitivity analysis is necessary for outlier detection in the network DEA models, including our models that are discussing cost efficiency analysis of network DEA models. However, we did not face such an issue in our data and our analysis. For more details see Table 3.

Table 3: Cost efficiency of Mashhad Hospital: ward and hospital level

DMUs and their processes	Conventional cost efficiency (CE_k)	Parallel cost efficiency (PCE)		
		CE_k^N	$w^{(p)}$	$w^{(p)}CE_k^{(p)}$
H1	**0.8712**	**0.0794**	**1.000**	**0.0794**
1. Operation		0.0663	0.6243	0.0414
2. Emergency		0.2880	0.1210	0.0348
3. ICU		0.0127	0.2547	0.0032
H2	**0.4443**	**0.0405**	**1.000**	**0.0405**
4. Operation		0.0311	0.5787	0.0180
5. Emergency		0.0901	0.2176	0.0196
6. ICU		0.0141	0.2037	0.0029
H3	**0.3403**	**0.0310**	**1.000**	**0.0310**
7. Operation		0.0202	0.8475	0.0171
8. Emergency		0.1693	0.0678	0.0115
9. ICU		0.0286	0.0847	0.0024
H4	**0.3603**	**0.0327**	**1.000**	**0.0327**
10. Operation		0.0320	0.7408	0.0237
11. Emergency		0.0423	0.1838	0.0078
12. ICU		0.0164	0.0754	0.0012
H5	**0.4967**	**0.0453**	**1.000**	**0.0453**
13. Operation		0.0323	0.5228	0.0169
14. Emergency		0.1416	0.1802	0.0255
15. ICU		0.0097	0.2970	0.0029
H6	**0.3436**	**0.0314**	**1.000**	**0.0314**
16. Operation		0.0290	0.6503	0.0189
17. Emergency		0.1475	0.0560	0.0083

Table 3: Cost efficiency of Mashhad Hospital: ward and hospital level

DMUs and their processes	Conventional cost efficiency (CE_k)	Parallel cost efficiency (PCE)		
		CE_k^N	$w^{(p)}$	$w^{(p)}CE_k^{(p)}$
18. ICU		0.0142	0.2937	0.0042
H7	**1**	**0.0912**	**1.000**	**0.0912**
19. Operation		0.0356	0.8601	0.0306
20. Emergency		1.000	0.0576	0.0576
21. ICU		0.0363	0.0823	0.0030
H8	**0.3672**	**0.0335**	**1.000**	**0.0335**
22. Operation		0.0264	0.6201	0.0164
23. Emergency		0.0822	0.1803	0.0148
24. ICU		0.0115	0.1996	0.0023
H9	**0.3539**	**0.0323**	**1.000**	**0.0323**
25. Operation		0.0413	0.6320	0.0261
26. Emergency		0.0202	0.2208	0.0045
27. ICU		0.0118	0.1472	0.0017
H10	**0.3556**	**0.0324**	**1.000**	**0.0324**
28. operation		0.0343	0.6245	0.0214
29. Emergency		0.0410	0.2105	0.0086
30. ICU		0.0146	0.1650	0.0024
H11	**0.2870**	**0.0263**	**1.000**	**0.0263**
31. Operation		0.0241	0.5739	0.0138
32. Emergency		0.0410	0.2783	0.0114
33. ICU		0.0078	0.1478	0.0011

In the first stage we performed a conventional and network technical efficiency analysis, ignoring price information. The results of this analysis is reported in Table 2. Ignoring the structure of hospitals and considering them as a black box, the average technical efficiency in hospitals was found to be 0.6, however, considering their internal structure, an average efficiency of 0.1 was observed in hospitals. It indicates the need for more improvements at hospital level. Nevertheless, the access to price information provides more information and yields a better insight toward the status of hospitals in terms of their cost efficiency.

Table 2: Technicalefficiency of Mashhad Hospital: ward and hospital level

DMUs and their processes	Conventional technical efficiency (E_k)	Parallel technical efficiency		
		$\boldsymbol{E_k^N}$ $E_k^{(p=1,2,3)}$	$\boldsymbol{w}$ $w^{(p=1,2,3)}$	$\boldsymbol{w\,E_k^N}$ $w^{(p=1,2,3)}E_k^{(p=1,2,3)}$
H1	**1.000**	**0.1932**	**1.000**	**0.1932**
1. Operation		0.1506	0.6680	0.1006
2. Emergency		1.000	0.0847	0.0847
3. ICU		0.0319	0.2473	0.0079
H2	**0.7037**	**0.1035**	**1.000**	**0.1035**
4. Operation		0.0779	0.5913	0.0461
5. Emergency		0.2085	0.2400	0.0500
6. ICU		0.0437	0.1687	0.0074

Table 2: Technicalefficiency of Mashhad Hospital: ward and hospital level

DMUs and their processes	Conventional technical efficiency (E_k)	Parallel technical efficiency		
		$\boldsymbol{E_k^N}$ $E_k^{(p=1,2,3)}$	$\boldsymbol{w}$ $w^{(p=1,2,3)}$	$\boldsymbol{w\ E_k^N}$ $w^{(p=1,2,3)}E_k^{(p=1,2,3)}$
H3	**0.4315**	**0.0785**	**1.000**	**0.0785**
7. Operation		0.0505	0.8588	0.0434
8. Emergency		0.4163	0.0706	0.0290
9. ICU		0.0868	0.0706	0.0061
H4	**0.5071**	**0.0925**	**1.000**	**0.0925**
10. Operation		0.0800	0.8381	0.0670
11. Emergency		0.1911	0.1150	0.0220
12. ICU		0.0744	0.0469	0.0035
H5	**0.6255**	**0.1004**	**1.000**	**0.1004**
13. Operation		0.0796	0.4700	0.0374
14. Emergency		0.1722	0.3286	0.0566
15. ICU		0.0319	0.2014	0.0064
H6	**0.4776**	**0.0790**	**1.000**	**0.0790**
16. Operation		0.0714	0.6682	0.0477
17. Emergency		0.1537	0.1356	0.0208
18. ICU		0.0538	0.1962	0.0105
H7	**1**	**0.1565**	**1.000**	**0.1565**
19. Operation		0.0678	0.7743	0.0525
20. Emergency		1.000	0.0989	0.0989
21. ICU		0.0404	0.1268	0.0051
H8	**0.5455**	**0.0793**	**1.000**	**0.0793**
22. Operation		0.0661	0.5872	0.0388
23. Emergency		0.1374	0.2558	0.0351
24. ICU		0.0347	0.1570	0.0054
H9	**0.5008**	**0.0904**	**1.000**	**0.0904**
25. Operation		0.1026	0.7110	0.0729
26. Emergency		0.0739	0.1690	0.0125
27. ICU		0.0416	0.1200	0.0050
H10	**0.4484**	**0.0851**	**1.000**	**0.0851**
28. operation		0.0681	0.8252	0.0562
29. Emergency		0.3593	0.0630	0.0226
30. ICU		0.0564	0.1118	0.0063
H11	**0.4081**	**0.0669**	**1.000**	**0.0669**
31. Operation		0.0592	0.5987	0.0354
32. Emergency		0.1065	0.2681	0.0285
33. ICU		0.0221	0.1332	0.0030

Classic and network efficiency analysis in hospitals are performed using the proposed model and the results are reported in Table 3. The second column shows the classic cost efficiency measure in hospitals. However, the network structure of hospitals is ignored in indices reported in this column. Considering both hospital and ward performances in the network structure, we measured parallel cost efficiency in hospitals and the results are reported in bold numbers in the third column. As presented, some hospitals are found to be cost efficient and cost inefficient using classic model and parallel cost model, respectively (see H1, H7 and H10 for instance). Moreover, as stated in proposition 2, more discriminating power is observed when using the PCE model, as compared with classical cost efficiency model. In other words, parallel cost efficiency values calculated by PCE model are smaller than or equal to those calculated by the conventional cost efficiency model.

Another important result is the cost efficiency values of wards reported by the numbers in the third column of Table 3. It shows not only the cost efficiency values of wards itself, but also enables decision makers to identify the source of internal cost inefficiency. For instance, one of the classic cost efficient hospitals like H7 is not cost efficient when measured using PCE model; however, it has a higher PCE score, as compared with other hospitals. This performance is more similar to its third ward i.e. ICU in terms of cost efficiency.

The fourth column of Table 3 presents the relative weights of each ward in each hospital $w^{(p)} = \frac{\sum_{i\in I(p)} v_{ik}^{*} X_{ik}^{(p)}}{\sum_{i=1}^{m} v_{ik}^{*} X_{ik}}$ and the fifth column presents associated terms in the decomposition equation of $CE_k^N = \sum_{p=1}^{q} w^{(p)} CE_k^{(P)}, \ \sum_{p=1}^{q} w^{(p)} = 1 , w^{(p)} \geq 0 , p = 1, \dots , q$. For instance, in H1 hospital cost efficiency is 0.36 that is computed by weighted sum of its ward cost efficiency, as follows:

$$0.0794 = 0.0663 * 0.6243 + 0.2880 * 0.1210 + 0.127 * 0.2547 = 0.0414 + 0.0348 + 0.0032$$

Generally, it indicates that the average hospital cost efficiency, as compared with the hospital system, is low, namely, about 0.32 while the average ward cost efficiency is higher than that, i.e. about 0.38, though it is still low. It highlights a serious need to pay attention to cost efficiency in hospitals in Mashhad, especially in the hospital level, as compared with ward level.

Among different wards, ICU ward was ranked the first level in terms of cost efficiency, and emergency ward and operation ward ranked the second and third, respectively.

Thus, more attention should be paid to cost minimization and cost management in hospitals in Mashhad.

Conclusion

The first stage for every improvement is to determine the current status of production system under evaluation. Hospitals are classified the top level of importance since they are one of the complex organizations that play a key role in the front line of health care in every community. Like other production systems, because of resource limitation, cost management and cost analysis are unavoidable in hospitals. Thus, it is important for decision maker to have an overview of costs in hospitals at different levels. For government (in case of public hospitals that are investigated in our case study) the cost analysis of *all* hospitals in country level is important. Hospital managers and decision maker must pay attention to cost analysis and consequently cost management within hospitals at ward level.

Proposed models in the present paper provide methodological foundation of the cost analysis not only for decision makers at the macro (country) level but also for managers and decision makers at the micro (hospital) level. Therefore, it helps policy makers to have a better, comprehensive, and deep insight toward the current status of hospitals that is necessary for future policy making and planning. This study developed a novel parallel cost efficiency model to measure the cost efficiency of the system and its processes and then we applied dual parallel model to use weights of inputs and outputs for calculating cost efficiencies of processes. The law of one price for an activity is considered in the proposed models. For this type of systems, the conventional cost efficiency model considers the system as a whole, while the PCE model considers each process when measuring cost efficiency of a system. The proposed models are being utilized for cost efficiency analysis in hospitals in Mashhad both at hospital level and ward level. The proposed models in the paper are a methodological development of cost efficiency analysis for parallel network models and can be used in any production system with a parallel network production system. The study of cost efficiency problem in more complex and composite production systems needs more theoretical investigation, which is left for the future research. The first step of future line of research is of course to deal with cost efficiency of production systems with series structure.

Competing interest:

The authors declare that there is no conflict of interest.

Authors' contributions:

Mgh introduced the methodology, wrote and revised the paper, and supervised the research. ShN contributed in the study design, data analysis, interpreting the results and editing the paper. ZH participated in writing and running models. GGT collected and cleaned the data and contributed in data analysis and editing.

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