Appendix

In this section the notations are presented followed by the mathematical formulation of the problem.

Notations

The notations which are used in mathematical model are as follows:

Indices:

i, i' : index for surgery

r : index for operating room

s, s' : index for surgeon and assistant surgeon

p : Index for time slot (15-minute units)

ω : scenario index

l : Index for surgeon's group number or the seniority levels of residents and fellowships

Inputs:

N	Total number of surgeries to be scheduled;
Nb	Total number of time slots;
$FR_{i,r}$	if surgery i can be done in OR r, it takes 1 ; otherwise, 0;
$FS_{i,s}$	if surgery i can be done by surgeon s according to chronologic curriculum plan, it takes 1; otherwise, 0;
$FAS_{i,s}$	If surgeon s can be the assistant for surgery i according to chronologic curriculum plan, it takes 1; otherwise, 0;
$IsNH_i$	If operation i needs an assistant surgeon, it takes 1; otherwise, 0;
$Kclean$	The duration of additional cleaning of OR after the surgery of an infected patient (time slots);
$infect_i$	If the patient of surgery i is infected, it takes 1; otherwise, 0;
$d_i(\omega)$	Duration of surgery i under scenario ω (time slots);
$p(\omega)$	Probability of scenario ω;
L	planned session length for each OR(time slots);
C^O	Per time-slot overtime cost of an OR;
C^{IR}	Per time-slot idle time cost of an OR;
$\bar{d}_i$	Mean duration of surgery i;
$AtN_{i,s}$	If operation i should be performed under the supervision of an attending *surgeon* s, it take 1;otherwise, 0
SAT_S	The surgeon s availability start time;
KG_s	The surgeon s group number;
$mean_l$	Average total operation time assigned to group number l daily;
Dev_l	Allowable deviation from mean(l)

$\omega \in \Omega$ represents the random outcome of the realized scenario. Given n surgeries, we obtain a random vector $\xi(\omega) = \{d_1(\omega), d_2(\omega), \ldots, d_n(\omega)\}$. we denote the finite support of $\xi(\omega)$ by Ξ where $\Xi \in \mathbb{R}^n$.

Decision variables:

$x_{i,r}$	binary decision variable denoting whether surgery i is allocated to OR r or not;
$z_{i,s}$	binary decision variable denoting whether surgery i is allocated to surgeon s or not;
$A_{i,s}$	binary decision variable denoting whether surgery i is allocated to assistant surgeon s or not;
$Rseq_{i,i',r}$	binary decision variable denoting whether surgery i precedes surgery i' in OR r or not (defined for $((i,i',r): i \neq i')$;
$Sseq_{i,i',s}$	binary decision variable denoting whether surgery i precedes surgery i' on surgeon s or not (defined for $((i,i',s): i \neq i')$;
$ST_i\ (\omega)$	Start time for surgery i under scenario ω;
$CT_i\ (\omega)$	Completion time for surgery i under scenario ω;
$Omax_r(\omega)$	The finish time of the last surgery of OR r under scenario ω;
$TWO_r(\omega)$	Total idle time of OR r under scenario ω;
$Over_r(\omega)$	Over time in OR r, with respect to session length L under scenario ω;
$SumOp_s$	Sum of average processing times assigned to surgeon s;
u_i	Positive auxiliary variable;

Mathematical formulation

The first stage model of SORS can be formulated as follows:

$$min \qquad Q(x,z,Rseq,Sseq) = E_\xi[Q(x,z,Rseq,Sseq,\xi(\omega))] \qquad (1)$$

$s.t.$

$$\sum_r x_{i,r} = 1 \qquad \forall i \qquad (2)$$

$$\sum_s z_{i,s} = 1 \qquad \forall i \qquad (3)$$

$$\sum_s A_{i,s} = \mathrm{IsNH}_i \qquad \forall i \qquad (4)$$

$$z_{i,s} + A_{i,s} \leq 1 \qquad \forall i,s \qquad (5)$$

$$x_{i,r} \ \leq FR_{i,r} \qquad \forall i,r \qquad (6)$$

$$z_{i,s} \ \leq FS_{i,s} \qquad \forall i,s \qquad (7)$$

$$A_{i,s} \leq FAS_{i,s} \qquad \forall i : \mathrm{IsNH}_i = 1 \qquad (8)$$

$$Rseq_{i,i',r} + Rseq_{i',i,r} \leq x_{i,r} \qquad \forall\ i,i',r : i' > i \qquad (9)$$

$$Rseq_{i,i',r} + Rseq_{i',i,r} \leq x_{i',r} \qquad \forall\ i,i',r : i' > i \qquad (10)$$

$$Rseq_{i,i',r} + Rseq_{i',i,r} \geq x_{i,r} + x_{i',r} - 1 \qquad \forall\ i,i',r : i' > i \qquad (11)$$

$$Sseq_{i,i',s} + Sseq_{i',i,s} \leq z_{i,s} + A_{i,s} \qquad \forall\ i,i',s : i' > i \qquad (12)$$

$$Sseq_{i,i',s} + Sseq_{i',i,s} \leq z_{i',s} + A_{i',s} \qquad \forall\, i,i',s : i' > i \qquad (13)$$
$$Sseq_{i,i',s} + Sseq_{i',i,s} \geq z_{i,s} + z_{i',s} + A_{i,s} + A_{i',s} - 1 \qquad \forall\, i,i',s : i' > i \qquad (14)$$
$$u_i - u_{i'} + n \times \left(\sum_r Rseq_{i,i',r} \right) \leq n - 1 \qquad \forall i, i' \neq i \qquad (15)$$
$$u_i - u_{i'} + n \times \left(\sum_s Sseq_{i,i',s} \right) \leq n - 1 \qquad \forall i, i' \neq i \qquad (16)$$
$$SumOp_s = \sum_i z_{i,s} \times \bar{d}_i \qquad \forall s \qquad (17)$$
$$SumOp_s - mean_l \geq -Dev_l \qquad \forall s, 1 \mid KG_s = 1 \qquad (18)$$
$$x_{i,r}, z_{i,s}, A_{i,s}, Rseq_{i,i',r}, Sseq_{i,i',s} \in \{0,1\} \quad , u_i \geq 0 \qquad \forall i, i' \neq i, r, s \qquad (19)$$

The objective function of the first stage is the expected second-stage cost over all scenarios (Eq.(1)). The second-stage cost is the sum of expected OR idle time cost and overtime cost. Eq. (2) and (3) ensure that each surgery is assigned to exactly one OR and one surgeon. Eq.(4) and (5) relate to assigning assistant surgeons to operations that require an assistant surgeon. Eq. (6) Forbids performing operation in a room where required equipment is not available. Eq. (7) and (8) specify that the operation should not be assigned to the surgeon and the assistant surgeon that are not qualified for it. These two constraints consider chronologic curriculum plan for training residents. Eq.(9)-(11) enforce that a precedence relation exists between two surgeries if and only if they are both assigned to the same OR. Also a precedence relation between two surgeries within each surgeon exists if and only if they are both assigned to the same surgeon (Eq.(12)-(14)). Eq.(15) and (16) are sub-tour breaking constraints for surgery sequences. Eq.(17) and (18) ensure the balanced distribution of operations between several surgeons.

Constraint (19) defines binary and non-negativity restrictions for the first-stage decision variables.

The second stage problem:

$$Q(x, z, Rseq, Sseq, \xi(\omega)) = \min \; C^{IR} \times \sum_r TWO_r(\omega) + C^O \times \sum_r Over_r(\omega) \qquad (20)$$

The constraints of the second-stage problem are as follows:

$$CT_i(\omega) - CT_{i'}(\omega) \geq d_i(\omega) - M \times \left(1 - \sum_r Rseq_{i',i,r} \right) \qquad \forall i, i' \neq i \qquad (21)$$
$$CT_i(\omega) - CT_{i'}(\omega) \geq d_i(\omega) - M \times \left(1 - Sseq_{i',i,s} \right) \qquad \forall i, i', s \; (i \neq i') \qquad (22)$$
$$CT_i(\omega) \geq d_i(\omega) \qquad \forall i \qquad (23)$$
$$ST_i(\omega) = CT_i(\omega) - d_i(\omega) + 1 \qquad \forall i \qquad (24)$$

$$ST_{i'}(\omega) - CT_i(\omega) \geq Kclean + 1 - M \times \left(1 - \sum_r Rseq_{i,i',r}\right) - M \times (1 - infect_i) - M \times infect_{i'} \qquad \forall i, i' | i' \neq i \qquad (25)$$

$$Omax_r(\omega) - CT_i(\omega) \geq -M \times (1 - x_{i,r}) \qquad \forall i, r \qquad (26)$$

$$TWO_r(\omega) - Omax_r(\omega) \geq -\sum_i x_{i,r} \times d_i(\omega) \qquad \forall r \qquad (27)$$

$$Over_r(\omega) \geq Omax_r(\omega) - L \qquad \forall r \qquad (28)$$

$$ST_i(\omega) \geq SAT_S - SAT_S \times \ (1 - z_{i,s}) \qquad \forall i, s \qquad (29)$$

$$ST_i(\omega) \geq SAT_S - SAT_S \times \ (1 - A_{i,s}) \qquad \forall i, s \qquad (30)$$

$$ST_i(\omega) \geq SAT_S - SAT_S \times \ (1 - ATN_{i,s}) \qquad \forall i, s | ATN_{i,s} = 1 \qquad (31)$$

$$CT_i(\omega), ST_i(\omega), Omax_r(\omega), TWO_r(\omega), Over_r(\omega) \geq 0 \qquad \forall \omega, i, r, p \qquad (32)$$

The objective function of the second-stage problem is formulated in Eq.(20). In Eq.(21)-(23) the completion time of surgeries considering their precedence relation on the OR and on the surgeon is defined. Eq.(21) shows that a surgery can be started in an OR when its previous surgery in the same OR is completed. The *M* parameter used in the constraints is an upper bound on completion time of the surgeries. The start time of surgeries is defined in Eq.(24) based on its completion time.

Constraint (25) is the infection prevention constraint. It states that when the surgery of an infected patient is scheduled for a specific period in an OR, the non-infected patient cannot be assigned to that OR until additional cleaning are performed. Constraint (26) determines the finish time of the last surgery of each operating room. Eq.(27) determines the total idle time of each OR between surgeries. The overtime used in each OR is defined in Eq.(28). Eq.(29-31) are related to the availability of surgeon, assistant surgeon and attends to start the procedure. The non-negativity and binary restrictions for the second-stage decision variables are defined in Eq.(32).